Mémoire présenté pour l’obtention du diplôme de Master en Informatique Industrielle

THEME

State Feedback Control of DC-DC Power Converters

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Abstract

This work investigates the state feedback control of certain DC-DC power converters. As a first step, the models of the considered converters are established; their properties are analyzed in both dynamic and static regimes. As a second step, the linear state feedback, based on the poles placement, is designed and tested on the modelled DC-DC converters. For the case of unavailable states, a linear full order observer is introduced to estimate the missing variables. As a third step, the passivity based control (PBC) is considered. The PBC exploits the energetic structure of the DC-DC converters to achieve the desired performance. Also, a passivity based observer is introduced to estimate the unmeasured variables. The simulations tests were conducted in the PSIM software environment. The obtained results were effective and confirm the theoretical predictions.
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Introduction

Power supply technology is an enabling technology that allows us to build and operate electronic circuits and systems. All active electronic circuits, both digital and analog, require power supplies. Many electronic systems require several DC supply voltages. Power supplies are widely used in computers, telecommunications, instrumentation equipment, aerospace, medical, and defense electronics. An DC supply voltage is usually derived from a battery or an ac utility line using a transformer, rectifier, and filter. The resultant DC voltage is not constant enough and contains a high ac ripple that is not appropriate for most applications. Voltage regulators are used to make the DC voltage more constant and to attenuate the ac ripple [1].

Power electronics devices are physical devices that can be mathematically modeled as controlled dynamic systems and, hence, they are suitably conformed for the application of existing control theories. Specifically, control theory is mainly concerned in the design of the regulating subsystem in a power electronics device for enhancing its overall performance in accordance with the prescribed objective. Although difficult, the objectives behind the design of a certain power electronics device can usually be translated into a rather concrete “control objective” for which an arsenal of techniques exist nowadays [2-3].

The basic aim of this work is to present two control techniques relevant to the design of feedback controllers for DC-DC power converters, namely, the poles placement based state feedback and passivity based control.

The present manuscript is organized as follows.

1. In the first chapter, we develop the switched models of the five DC-DC converters under study. Since we are using continuous-time control techniques, the switched models are converted into continuous averaged models. The study of the steady state averaged dynamic permits the assessment of the converters input–output behaviors. Linearized models are established and their proprieties analyzed for the linear state feedback purpose.

2. In chapter two, the linearized models, from the first chapter, are used for the design of linear state feedback control of the DC-DC converters, in order to achieve a particular desired behavior. To guarantee a zero steady state error, we introduce an integral action, which will work out this problem by assuring that the steady state error will end up to zero. If it is supposed that both the voltage(s) and current(s) are measured, so much more sensors are needed then and consequently causing a high cost, so that to estimate the current(s) with a low cost and less complexity it is preferred to introduce a state observer, considering that the voltage(s) is the only measurable variable(s).

3. The passivity based control (PBC) approach has been also applied to DC-DC converters, and then a passivity based non-linear observer will be used for the estimation of the state variables. The PBC exploits useful energetic properties of the DC-DC converters to get a simpler and stable control structures.

The conclusion gives some final remarks on the realized work.
1.1. Introduction

A fundamental step for the implementation of control techniques is the representation of converter in a form of dynamic system, with an appropriate model for the proposed control technique. This chapter will be focused on the modeling of the DC-DC converters for the application of the selected control techniques. In what follows, it is assumed that the DC-DC converters operate in the continuous conduction mode, i.e. neither of the inductor currents are identically zero on an open interval of time.

The modeling has to go through the following steps:

1. Determine the switched or the topological converter model. The switched model usually takes the following form:

   \[ \dot{x} = f(x, u) \]
   \[ y = Cx \]

   With \( u \in \{0, 1\} \) is the switching function (or switching functions, according to the number of the independent switches), \( x \in R^n \) is the state vector (composed of the inductors currents and capacitors voltages), and \( y \) is the output (or outputs) that will be controlled.

2. Obtain the average model (that is almost always non-linear). This is realized by replacing the switching function \( u \) by its average value \( u_{av} \in [0, 1] \), which yields the average model as follows:

   \[ \dot{x} = f(x, u_{av}) \]
   \[ y = Cx \]

3. Define the constant desired operating values (current, voltage) around on which we want to control the converter, this process is the action of fixing the constant desired values for the output \( y \) (usually the reference value for the output voltage(s); then resolve the average model equations within the steady state regime:

   \[ 0 = f(\bar{x}, \bar{u}_{av}) \]
   \[ \bar{y} = C\bar{x} \]

   To find back the reference values for the remaining states and control inputs \( (\bar{x}, \bar{u}_{av}) \)

4. Once we have obtained the average model, we proceed to perform the linearization of the model around the desired equilibrium point. The linearization of the average model (1.2), around the desired equilibrium point \( (\bar{x}, \bar{u}_{av}) \), yields the following state equations:

   \[ \dot{x} = A\dot{x} + B\bar{u}_{av} \]
   \[ \bar{y} = C\bar{x} \]

   With \( \tilde{x} = x - \bar{x}, \bar{u}_{av} = u_{av} - \bar{u}_{av}, \bar{y} = y - \bar{y} \), and the matrices \( A \) and \( B \) are obtained via the Taylor first order development.
1.2 Buck converter

The Buck converter is used for step down operation. A DC-DC buck converter with its output filter arrangement is shown in figure 1.1.

Fig. 1.1 Practical Buck converter realization.

### 1.2.1 Switched Converter model

In order to obtain the Buck converter switched model, all we have to do is to apply the Kirchhoff’s laws to fig. 1.1 circuit, and to combine the different topologies (fig. 1.2) into a switched state-space model.

The two states are given by:

![Diagram of Buck converter states](image)

Fig.1.2 (a) $u = 1$, (b) $u = 0$.

When the ideal switch is ON as shown in the figure 1.2 (a) the dynamics of the inductor current $i(t)$ and the capacitor voltage $v(t)$ are both given by:

\[
\begin{align*}
L \frac{di}{dt} &= -v + E \\
C \frac{dv}{dt} &= i - \frac{v}{R}
\end{align*}
\]  

(1.1)

And when the switch is OFF as shown in the figure 1.2 (b) the dynamics are presented by:

\[
\begin{align*}
L \frac{di}{dt} &= -v \\
C \frac{dv}{dt} &= i - \frac{v}{R}
\end{align*}
\]  

(1.2)

After doing a comparison between the two states (ON and OFF) we get the following unified model:
Chapter 1                                                                                              Open-loop modeling of DC-DC converters

\[
\begin{align*}
L \frac{di}{dt} & = -v + uE \\
C \frac{dv}{dt} & = i - \frac{v}{R}
\end{align*}
\]  

(1.3)

So, when \( u = 1 \) or \( u = 0 \) we find back the model (1.1) or the model (1.2). The model (1.3) is often called switched model with the binary function of the switching \( u \in \{0, 1\} \).

1.2.2 Average model

The average converter model is exactly the same as (1.3), except that the control variable \( u \) is replaced by its continuous average variable \( u_{av} \) that takes its values in the interval \([0, 1]\).

The average model of the Buck converter is found to be given by:

\[
\begin{align*}
L \dot{x}_1 & = -x_2 + u_{av}E \\
C \dot{x}_2 & = x_1 - \frac{x_2}{R}
\end{align*}
\]  

(1.4)

With: \( x_1 = i, x_2 = v \)

In the matrix form, we get:

\[
\dot{x} = Ax + Bu_{av}.
\]

\[
\dot{x} = \begin{bmatrix}
0 & -1/L \\
1/C & -1/RC
\end{bmatrix} x + \begin{bmatrix}
E/L \\
0
\end{bmatrix} u_{av}
\]  

(1.5)

The average model is obviously linear; in addition it is controllable and observable for each of the two output states.

1.2.3 Equilibrium point

The main purpose of the control will be often to regulate the output voltage to a desired average value. This is fulfilled by choosing the input \( u \) that will control the switch state (position) according to the average value (reference), which is interpreted as the average duty cycle in PWM techniques, and can be interpreted as an equivalent control.

Generally, within equilibrium, it is a likely linking together, the average values of the system state, and the constant average values corresponding to the control input.

These relations, within the equilibrium state, are useful in defining the static converter characteristics.

In equilibrium state, the derivation of the mean (average) states is zero, and the average control input \( u_{av} \) takes a constant value \( \bar{u}_{av} \). Then, using the representation (1.4) and noting the average equilibrium as \( \bar{x}_1 \) and \( \bar{x}_2 \), we get at equilibrium:

\[
\begin{align*}
0 & = -\bar{x}_2 + \bar{u}_{av}E \\
0 & = \bar{x}_1 - \frac{\bar{x}_2}{R}
\end{align*}
\]  

(1.6)

After solving the system equations (1.6), we get the equilibrium states of the system as next:
This parameterization of the equilibrium point by the average control is useful to determine the converter damping character.

Relation (1.7) shows that the average output voltage is a fraction of $E$, and the converter cannot amplify the input voltage since $\bar{u}_{av}$ is restricted in $[0, 1]$.

![Fig. 1.3: Static transfer function](image)

1.2.4 Desired equilibrium point

Supposing that the desired average voltage at equilibrium is $\bar{x}_2 = EV_d$, so we get at equilibrium:

$$\bar{x}_1 = \frac{EV_d}{R}, \quad \bar{x}_2 = EV_d, \quad \bar{u}_{av} = V_d$$

Hence, the desired average voltage has to satisfy $0 < V_d < 1$.

1.3 Boost converter

The boost converter has earned its name grace to its ability of producing an DC output voltage greater in magnitude than the DC input voltage (step-up). The circuit topology for the boost converter is as shown in figure 1.4

![Fig. 1.4 Practical Boost converter realization](image)

When the transistor Q is ON (fig.1.5 (a)), The current in inductor L, rises linearly and at this time capacitor C, supplies the load current, and it is partially discharged. During the second interval when transistor Q is OFF (fig.1.5 (b)). The diode D is ON and the inductor L, supplies the load and, additionally, recharges the capacitor C.
1.3.1. Switched Converter model:

When the switching function is \( u = 1 \), the following dynamic is obtained:

\[
L \frac{di}{dt} = E \\
C \frac{dv}{dt} = -\frac{v}{R}
\]  

When the switching function is \( u = 0 \), the following dynamic is obtained:

\[
L \frac{di}{dt} = -v + E \\
C \frac{dv}{dt} = i - \frac{v}{R}
\]

So the dynamic of the converter is as follows:

\[
L \frac{di}{dt} = -(1 - u)v + E \\
C \frac{dv}{dt} = (1 - u)i - \frac{v}{R}
\]

1.3.2 Average model

So, we get the following unified average model:

\[
L A \frac{d\bar{x}}{dt} = -(1 - u A)\bar{x} + E \\
C \frac{d\bar{y}}{dt} = (1 - u A)\bar{x} - \frac{\bar{y}}{R}
\]

Or:

\[
L \bar{x}_1 = -(1 - u A)\bar{x}_2 + E \\
C \bar{x}_2 = (1 - u A)\bar{x}_1 - \frac{\bar{x}_2}{R}
\]

With \( \bar{x}_1 = i, \bar{x}_2 = v \)

The variable \( \bar{x}_1 \) is the average current of the inductor and \( \bar{x}_2 \) is the output average voltage.

1.3.3 Equilibrium point

In equilibrium state, the derivation of the mean (average) states is zero, and the average control \( u_A \) takes a constant value \( u_{av} \), as a result, we get the linear system equations for the
values in steady state of average states, noting the equilibrium average values of the current and voltage as $\bar{x}_1$ and $\bar{x}_2$, so we get at equilibrium:

$$ \begin{bmatrix} 0 & 1 - \bar{u}_{av} \\ 1 - \bar{u}_{av} & -\frac{1}{R} \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{1.14} $$

After solving the system equations (1.14), we get the system equilibrium states as follows:

$$ \bar{x}_1 = \frac{E}{R} \frac{1}{(1 - \bar{u}_{av})^2}, \quad \bar{x}_2 = \frac{E}{(1 - \bar{u}_{av})} \tag{1.15} $$

Fig. 1.6: Static transfer function.

1.3.4 Desired equilibrium point

The average model of this converter is found to be given by:

$$ L \dot{x}_1 = -(1 - u_{av})x_2 + E $$
$$ C \dot{x}_2 = (1 - u_{av})x_1 - \frac{x_2}{R} \tag{1.16} $$

If the desired average voltage at equilibrium is $\bar{x}_2 = EV_d$, then, we get at equilibrium:

$$ \bar{x}_1 = \frac{E}{R} V_d^2, \quad \bar{u}_{av} = 1 - \frac{1}{V_d} \tag{1.17} $$

As if $\frac{E}{R} > 0$ then $\bar{x}_1 > 0$ and the desired average voltage has to satisfy $1 < V_d < \infty$.

1.3.5 Linearization

The linearization of the average model is found to be given by:

$$ L \ddot{x}_1 = -\frac{1}{V_d} \dot{x}_2 + EV_d \bar{u}_{av} \tag{1.18} $$
$$ C \ddot{x}_2 = \frac{1}{V_d} \ddot{x}_1 - \frac{1}{R} \dot{x}_2 - \frac{E}{R} V_d^2 \bar{u}_{av} $$

With

$$ \ddot{x}_1 = x_1 - \bar{x}_1 $$
\[
\dot{x}_2 = x_2 - \bar{x}_2 \\
\dot{u}_{av} = u_{av} - \bar{u}_{av}
\] (1.19)

In the matrix form, we get:
\[
\dot{x} = \begin{bmatrix}
0 & -1/LV_d \\
1/CV_d & -1/RC
\end{bmatrix} x + \begin{bmatrix}
EV_d/L \\
EV_d^2/RC
\end{bmatrix} \dot{u}_{av}
\] (1.20)

The linearized model is controllable as being proved next:
\[
M = \begin{bmatrix}
EV_d/L & EV_d/RLC \\
-EV_d^2/RC & E + EV_d/RCL
\end{bmatrix}, \quad \text{det } M = \frac{EV_d}{L(RC)^2}(R^2C + LV_d + V_d^2) \neq 0
\] (1.21)

Our model is also observable for the two states variables

For \( \ddot{y} = \ddot{x}_1 \), comes up
\[
N = \begin{bmatrix}
1 & 0 \\
0 & -1/LV_d
\end{bmatrix}, \quad \text{det } N = -\frac{1}{LV_d} \neq 0
\] (1.22)

For \( \ddot{y} = \ddot{x}_2 \), we get then:
\[
N = \begin{bmatrix}
0 & \frac{1}{CV_d} \\
1 & -1/RC
\end{bmatrix}, \quad \text{det } N = -\frac{1}{CV_d} \neq 0
\] (1.23)

### 1.4 Buck-Boost Converter

The buck-boost converter is capable of producing a DC output voltage which is either greater or smaller in magnitude than the DC input voltage. The arrangement for the buck-boost converter is as shown in figure (1.7)

![Fig.1.7 Practical Buck-Boost converter realization](image)

When the transistor Q is ON fig (1.8(a)), input voltage is applied across the inductor and the current in inductor L rises linearly. At this time the capacitor \( C \), supplies the load current, and it is partially discharged. During the second interval when the transistor is OFF fig.1.8 (b), the voltage across the inductor reverses in polarity and the diode conducts. During this interval the energy stored in the inductor supplies the load and, additionally, recharges the capacitor.
1.4.1. Switched Converter model

By applying the Kirchoff’s laws on the two previous circuits, we get the following dynamics:

When the switching function is \(u=1\), the following dynamic is obtained:

\[
L \frac{di}{dt} = E \\
C \frac{dv}{dt} = -\frac{v}{R}
\]  

When the switching function is \(u=0\), the following dynamic is obtained:

\[
L \frac{di}{dt} = v \\
C \frac{dv}{dt} = -i - \frac{v}{R}
\]  

So the dynamic of the converter is found to be given as:

\[
L \frac{di}{dt} = (1 - u)v + uE \\
C \frac{dv}{dt} = -(1 - u)i - \frac{v}{R}
\]  

1.4.2 Average model

The average model of the Buck-Boost converter is found to be given as:

\[
L \dot{x}_1 = (1 - u_{av})x_2 + u_{av}E \\
C \dot{x}_2 = -(1 - u_{av})x_1 - \frac{x_2}{R}
\]  

With \(x_1 = i, x_2 = v\)

1.4.3 Equilibrium point

At equilibrium, we get:

\[
\begin{bmatrix}
0 & (1 - \bar{u}_{av}) \\
-(1 - \bar{u}_{av}) - 1/R & 1
\end{bmatrix}
\begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2
\end{bmatrix}
= 
\begin{bmatrix}
-E\bar{u}_{av} \\
0
\end{bmatrix}
\]

After solving the system equations (1.28), we get the system equilibrium states as follows:

\[
\bar{x}_1 = \frac{E}{R (1 - \bar{u}_{av})^2}, \bar{x}_2 = -\frac{E\bar{u}_{av}}{(1 - \bar{u}_{av})}
\]
1.4.4 Desired equilibrium point

The average model of this converter is found to be described by:

\[
L\dot{x}_1 = (1 - u_{av})x_2 + Eu_{av} \\
C\dot{x}_2 = -(1 - u_{av})x_1 - \frac{x_2}{R}
\]

(1.30)

If the desired average voltage at equilibrium is \( \bar{x}_2 = EV_d \), so we have at equilibrium:

\[
\bar{x}_1 = \frac{EV_d(V_d - 1)}{R}, \bar{u}_{av} = \frac{V_d}{V_d - 1}
\]

(1.31)

As \( \frac{E}{R} > 0 \) then \( \bar{x}_1 > 0 \) and the desired average voltage has to satisfy \( V_d < 0 \).

1.4.5 Linearization

The average model linearization through the equilibrium points is as below:

\[
\begin{align*}
\dot{x}_1 &= \frac{-1}{L(V_d - 1)} \bar{x}_2 + \frac{E(1 - V_d)}{L} \bar{u}_{av} \\
\dot{x}_2 &= \frac{1}{C(V_d - 1)} \bar{x}_1 - \frac{1}{RC} \bar{x}_2 + \frac{EV_d(V_d - 1)}{RC} \bar{u}_{av}
\end{align*}
\]

(1.32)

With:

\[
\begin{align*}
\bar{x}_1 &= x_1 - \bar{x}_1 \\
\bar{x}_2 &= x_2 - \bar{x}_2 \\
\bar{u}_{av} &= u_{av} - \bar{u}_{av}
\end{align*}
\]

(1.33)

In the matrix form, we get:

\[
\dot{\bar{x}} = \begin{bmatrix}
0 & \frac{-1}{L(V_d - 1)} \\
\frac{1}{C(V_d - 1)} & -\frac{1}{RC}
\end{bmatrix} \bar{x} + \begin{bmatrix}
\frac{E(1 - V_d)}{L} \\
\frac{EV_d(V_d - 1)}{RC}
\end{bmatrix} \bar{u}_{av}
\]

(1.34)

The controllability of the linearized model is fulfilled for the reason that:
Chapter 1 Open-loop modeling of DC-DC converters

\[ M = \begin{bmatrix} \frac{E(1-V_d)}{L} & -\frac{E V_d}{R L C} \\ \frac{E V_d(V_d-1)}{R C} & -\frac{E}{C L} - \frac{E V_d(1-V_d)}{(R C)^2} \end{bmatrix}, \det(M) = \frac{E^2(V_d-1)}{C L} \left( \frac{V_d}{R^2 C} + \frac{1}{C} \right) \neq 0 \quad (1.35) \]

In the other hand our model is also observable for all the two states variables

We get for \( \tilde{y} = \tilde{x}_1 \):

\[ N = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{L(V_d-1)} \end{bmatrix}, \quad \det N = -\frac{1}{L(V_d-1)} \neq 0 \quad (1.36) \]

The following is gotten for \( \tilde{y} = \tilde{x}_2 \):

\[ N = \begin{bmatrix} 0 & \frac{-1}{C(V_d-1)} \\ 1 & \frac{-1}{R C} \end{bmatrix}, \quad \det N = \frac{1}{C(V_d-1)} \neq 0 \quad (1.37) \]

1.5 Cuk converter

Cuk converter is derived from the cascading of buck and boost converters. The buck, boost and buck-boost converters all transfer energy between input and output using the inductor and analysis is based on voltage balance across the inductor. This converter is shown in fig. 1.10. The input circuit in the Cuk converter is clearly a Boost circuit converter, and the output circuit is seen to be a Buck converter. Thus, we may also think of the Cuk converter as a “Boost-Buck” converter.

![Cuk converter diagram](image)

Fig.1.10 Practical Cuk converter realization.

The Cuk converter operates in two steps. The first one is obtained when the transistor is ON in the same time the diode D is reversely polarized giving the circuit topology in fig. 1.11(a). During this period, the inductor current \( i_1 \) is drawn from the voltage source \( E \). This mode represents the charging mode. The second step starts up when the transistor is OFF and the diode D is directly polarized giving the circuit topology in fig. 1.11(b). This mode of operation is known as the discharging mode since all the energy stored in \( L_1 \) is now transferred to the load \( R \).
1.5.1 Switched Converter model

The derivation of the dynamics of the Cuk converter is carried out in the same manner in which we analyzed the topologies of the previous basic DC-DC power converters.

When $u = 1$, we obtain the following equations for $i_1$ and $i_2$ in the obtained circuit topology.

$$L_1 \frac{di_1}{dt} = E$$

$$L_2 \frac{di_2}{dt} = -v_1 - v_2$$

And the following equations for the capacitor voltages $v_1$ and $v_2$,

$$C_1 \frac{dv_1}{dt} = i_2$$

$$C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R}$$

When $u = 0$, we obtain the following equations for $i_1$ and $i_2$.

$$L_1 \frac{di_1}{dt} = -v_1 + E$$

$$L_2 \frac{di_2}{dt} = -v_2$$

The capacitor voltages $v_1$ and $v_2$ are described by:

$$C_1 \frac{dv_1}{dt} = i_1$$

$$C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R}$$

The Cuk switched model is then given by combining the previous partial models, that is:

$$L_1 \frac{di_1}{dt} = -(1 - u)v_1 + E$$

$$C_1 \frac{dv_1}{dt} = (1 - u)i_1 + ui_2$$

$$L_2 \frac{di_2}{dt} = -uv_1 - v_2$$

$$C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R}$$
1.5.2 Average model

Replacing $u$ by $u_{av}$, we get the following average model:

$$L_1 \dot{x}_1 = -(1 - u_{av})x_2 + E$$

$$C_1 \dot{x}_2 = (1 - u_{av})x_1 + u_{av}x_3$$

$$L_2 \dot{x}_3 = -u_{av}x_2 - x_4$$

$$C_2 \dot{x}_4 = x_3 - \frac{x_4}{R}$$

With: $x_1 = \dot{i}_1, x_2 = v_1, x_3 = \dot{i}_2, x_4 = v_2$

(1.43)

1.5.3 Equilibrium point

At equilibrium, with the average control $u_{av}$ taking a constant value $\overline{u}_{av}$, we get as a result, the system equations for the steady state regime:

$$\begin{bmatrix}
0 & (1 - \overline{u}_{av}) & 0 & 0 \\
(1 - \overline{u}_{av}) & 0 & \overline{u}_{av} & 0 \\
0 & -\overline{u}_{av} & 0 & -1 \\
0 & 0 & 1 & -1/R
\end{bmatrix}\begin{bmatrix}
\overline{x}_1 \\
\overline{x}_2 \\
\overline{x}_3 \\
\overline{x}_4
\end{bmatrix} = \begin{bmatrix}
E \\
0 \\
0 \\
0
\end{bmatrix}$$

(1.44)

After solving the system equations (1.44), we get the system equilibrium states as follows:

$$\overline{x}_1 = \frac{E \overline{u}_{av}^2}{R (1 - \overline{u}_{av})^2}, \overline{x}_2 = \frac{E}{(1 - \overline{u}_{av})}, \overline{x}_3 = -\frac{E}{R (1 - \overline{u}_{av})} \overline{u}_{av}, \text{ and } \overline{x}_4 = -\frac{E \overline{u}_{av}}{(1 - \overline{u}_{av})}$$

(1.45)

As $E > 0$ then $\overline{x}_1 > 0, \overline{x}_3 < 0$ and the desired average voltage has to satisfy $V_d < 0$. 

1.5.4 Desired equilibrium point

If the desired average voltage at equilibrium is $\overline{x}_1 = EV_d$, so we have at equilibrium:

$$\begin{bmatrix}
\overline{x}_1 \\
\overline{x}_2 \\
\overline{x}_3 \\
\overline{x}_4
\end{bmatrix} = \begin{bmatrix}
E V_d^2 \\
E(1 - V_d) \\
E V_d \\
V_d / (V_d - 1)
\end{bmatrix}$$

(1.46)
1.5.5 Linearization

The linearization of the model is found to be given as below:

\[
\tilde{x}_1 = \frac{1}{L_1(V_d - 1)} \tilde{x}_2 + \frac{E(1-V_d)}{L_1} \tilde{u}_{\text{av}}
\]

\[
\tilde{x}_2 = -\frac{1}{C_1(V_d - 1)} \tilde{x}_1 + \frac{V_d}{C_1(V_d - 1)} \tilde{x}_3 + \frac{E V_d}{R C_1} \tilde{u}_{\text{av}}
\]

\[
\tilde{x}_3 = -\frac{V_d}{L_2(V_d - 1)} \tilde{x}_2 - \frac{1}{L_2} \tilde{x}_1 + \frac{E(V_d - 1)}{L_2} \tilde{u}_{\text{av}}
\]

\[
\tilde{x}_4 = \frac{1}{C_2} \tilde{x}_3 - \frac{1}{R C_2} \tilde{x}_4
\]

In which: \( \tilde{x}_1 = x_1 - \overline{x}_1 \), \( \tilde{x}_2 = x_2 - \overline{x}_2 \), \( \tilde{x}_3 = x_3 - \overline{x}_3 \), \( \tilde{x}_4 = x_4 - \overline{x}_4 \), \( \tilde{u}_{\text{av}} = u_{\text{av}} - \overline{u}_{\text{av}} \).

In the matrix form, we get:

\[
\dot{\tilde{x}} = \begin{bmatrix}
0 & 1/L_1(V_d-1) & 0 & 0 \\
-1/C_1(V_d-1) & 0 & V_d/C_1(V_d-1) & 0 \\
0 & -V_d/L_2(V_d-1) & 0 & -1/L_2 \\
0 & 0 & 1/C_2 & -1/R C_2 \\
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\tilde{x}_3 \\
\tilde{x}_4 \\
\end{bmatrix}
+ \begin{bmatrix}
E(1-V_d)/L_1 \\
EV_d/RC_1 \\
0 \\
-E(1-V_d)/L_2 \\
\end{bmatrix}
\tilde{u}_{\text{av}} \tag{1.48}
\]

On the one hand we got \( \det(M) \neq 0 \) so our model is controllable, in which that:

\[
M = \begin{bmatrix}
E(1-V_d)/L_1 & EV_d/(V_d-1)RLC_1 & (E/L_1)(V_d-1)((1+\frac{1}{L_1})V_d) & -(E/V_d/L_1)(\frac{1}{L_1} + \frac{1}{L_2})V_d/C_1(V_d) \\
EV_d/RC_1 & (E/C_1)(\frac{1}{L_1} + \frac{1}{L_2})V_d & -(EV_d/RC_1)(\frac{1}{L_1} + \frac{1}{L_2})V_d/C_1(V_d)^2 & -(EV_d/L_1)(\frac{1}{L_1} + \frac{1}{L_2})V_d/C_1(V_d) + \frac{1}{L_2}C_2 \\
-E(1-V_d)/L_1 & -EV_d/(V_d-1)RLC_1 & -(E/L_1)(V_d-1)((1+\frac{1}{L_1})V_d) + \frac{1}{L_1} & EV_d/(L_1C_1(V_d-1)) + \frac{V_d^2}{C_1(V_d-1)} + \frac{1}{L_2} - (E/V_d/L_1)RLC_2 \\
0 & -E(1-V_d)/L_1C_1 & (E/L_1C_1(V_d-1))((1+\frac{1}{L_1})V_d) - \frac{1}{L_1} & EV_d/(L_1C_1(V_d-1)) - \frac{V_d^2}{C_1(V_d-1)} - (E/V_d/L_1)RLC_2 \\
\end{bmatrix} \neq 0 \tag{1.49}
\]

On the other hand we got \( \det(N) \neq 0 \) for all the two mean state variables the model we go \( t \) is observable, given that:

For \( \tilde{y} = \tilde{x}_2 \) comes

\[
N = \begin{bmatrix}
0 & -1/L_1(V_d-1) & 0 & -\left(\frac{1}{L_1} + \frac{1}{L_2}\right)V_d^2/L_1C_1(1-V_d)^3 \\
1 & 0 & -\frac{V_d}{(1-V_d)^2C_1}\left(\frac{1}{L_1} + \frac{V_d^2}{L_2}\right) & 0 \\
0 & -V_d/L_2(V_d-1) & 0 & \frac{V_d}{C_1L_2(V_d-1)}(\frac{L_2}{L_1(V_d-1)} + \frac{V_d^2}{(1-V_d)^2}) + 1 \\
0 & 0 & -V_d/L_2(C_1(V_d-1)) & \frac{V_d}{RLC_2(V_d-1)} \\
\end{bmatrix} \neq 0 \tag{1.50}
\]

For \( \tilde{y} = \tilde{x}_4 \) comes
1.6 Zeta converter

The Zeta converter can both amplify and reduce, without polarity inversions, the value of the input source voltage $E$. We briefly summarize next the most important features involved in the modeling of the Zeta converter. Figure 1.13 depicts a semiconductor realization of a Zeta DC-to-DC power converter. The ideal switch based realization of the Zeta converter is depicted in Figure 1.13.

![Figure 1.13 The Zeta DC-DC converter realization](image)

The Zeta converter operates in two steps. The first one is obtained when the transistor is ON and instantaneously, the diode $D$ is reversely polarized giving the circuit shown in fig. 1.14(a). During this period, the inductors currents $i_1$ and $i_2$ are drawn from the voltage source $E$. This mode is the charging mode. The second mode starts up when the transistor is OFF and the diode $D$ is directly polarized giving the circuit shown in fig. 1.14(b). This mode of operation is known as the discharging mode, i.e., all the energy stored in $L_2$ will be transferred now to the load $R$.

![Fig 1.14 (a) $u = 1$, (b) $u = 0$.](image)

1.6.1 Switched Converter model

By applying the Kirchoff’s laws on the two previous circuits, we get the following dynamics:

When the switching function is $u=1$, the following dynamic is obtained:
\[ L_1 \frac{di_1}{dt} = E \]
\[ C_1 \frac{dv_1}{dt} = -i_2 \]
\[ L_2 \frac{di_2}{dt} = v_1 - v_2 + E \]
\[ C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R} \]

When the switching function is \( u = 1 \), the following dynamic is obtained:

\[ L_1 \frac{di_1}{dt} = -v_1 \]
\[ C_1 \frac{dv_1}{dt} = i_1 \]
\[ L_2 \frac{di_2}{dt} = -v_2 \]
\[ C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R} \]

The Zeta converter dynamic is then described by combining the previous partial models. We obtain the following system of differential equations:

\[ L_1 \frac{di_1}{dt} = -(1 - u)v_1 + uE \]
\[ C_1 \frac{dv_1}{dt} = (1 - u)i_1 - u i_2 \]
\[ L_2 \frac{di_2}{dt} = uv_1 - v_2 + uE \]
\[ C_2 \frac{dv_2}{dt} = i_2 - \frac{v_2}{R} \]

1.6.2 Average model

Replacing \( u \) by \( u_{av} \), we get the following average model:

\[ L_1 \dot{x}_1 = -(1 - u_{av})x_2 + u_{av}E \]
\[ C_1 \dot{x}_2 = (1 - u_{av})x_1 - u_{av}x_3 \]
\[ L_2 \dot{x}_3 = u_{av}x_2 - x_4 + u_{av}E \]
\[ C_2 \dot{x}_4 = x_3 - \frac{x_4}{R} \]

With: \( x_1 = i_1, x_2 = v_1, x_3 = i_2, x_4 = v_2 \)
1.6.3 Equilibrium point

Within the equilibrium state, with \( u_{av} = \bar{u}_{av} \), we get:

\[
\begin{bmatrix}
0 & -(1 - \bar{u}_{av}) & 0 & 0 \\
(1 - \bar{u}_{av}) & 0 & -\bar{u}_{av} & 0 \\
0 & \bar{u}_{av} & 0 & -1 \\
0 & 0 & 1 & -1/R
\end{bmatrix}
\begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2 \\
\bar{x}_3 \\
\bar{x}_4
\end{bmatrix}
= - \begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix} E
\]

(1.56)

After solving the system equations (1.59), we get the system equilibrium states as follows:

\[
\bar{x}_1 = E \frac{\bar{u}_{av}^2}{R (1 - \bar{u}_{av})^2}, \bar{x}_2 = E \frac{\bar{u}_{av}}{(1 - \bar{u}_{av})}, \bar{x}_3 = E \frac{\bar{u}_{av}}{R (1 - \bar{u}_{av})} \text{ and } \bar{x}_4 = E \frac{\bar{u}_{av}}{(1 - \bar{u}_{av})}
\]

(1.57)

![Fig. 1.15 Static transfer function](image)

1.6.4 Desired equilibrium point

If the desired average voltage at equilibrium is \( \bar{x}_4 = EV_d \), then we get:

\[
\bar{x}_1 = E \frac{V_d^2}{R}, \bar{x}_2 = EV_d, \bar{x}_3 = E \frac{V_d}{R} \text{ et } \bar{x}_{av} = \frac{V_d}{V_d + 1}
\]

(1.58)

As \( \frac{E}{R} > 0 \) then \( \bar{x}_1 > 0, \bar{x}_3 > 0 \) and the desired average voltage has to satisfy \( V_d > 0 \).

1.6.5 Linearization

The linearization of the model is found to be given as below:

\[
L_1 \dot{\bar{x}}_1 = \frac{1}{V_d + 1} \bar{x}_2 + E(1 + V_d)\bar{u}_{av}
\]

\[
C_1 \dot{\bar{x}}_2 = -\frac{1}{V_d + 1} \bar{x}_1 + \frac{V_d}{V_d + 1} \bar{x}_3 + \frac{V_d(1 + V_d)}{R} \bar{u}_{av}
\]

(1.59)

\[
L_2 \dot{\bar{x}}_3 = \frac{-V_d}{V_d + 1} \bar{x}_2 - \bar{x}_1 + E(V_d + 1)\bar{u}_{av}
\]

\[
C_2 \dot{\bar{x}}_4 = \bar{x}_3 - \frac{\bar{x}_4}{R}
\]
With: \( \bar{x}_1 = x_1 - \bar{x}_1, \bar{x}_2 = x_2 - \bar{x}_2, \bar{x}_3 = x_3 - \bar{x}_3, \bar{x}_4 = x_4 - \bar{x}_4 \), \( \bar{u}_{av} = u_{av} - \bar{u}_{av} \).

In the matrix form, we have:

\[
\dot{\bar{x}} = \begin{bmatrix}
0 & 1/L_i(V_d + 1) & 0 & 0 \\
-1/C_i(V_d + 1) & 0 & V_i / C_i(V_d + 1) & 0 \\
0 & -V_i / L_i(V_d + 1) & 0 & -1/L_i \\
0 & 0 & 1/C_2 & -1/RC_2
\end{bmatrix} \bar{x} + \begin{bmatrix}
E(1 + V_i) / L_i \\
E(V_d(1 + V_i))/RC_i \\
E(V_d(1 + V_i))/L_2 \\
0
\end{bmatrix} \bar{u}_{av}
\]  

(1.60)

It happened that the linearized model is controllable according to the following:

\[
M = \begin{bmatrix}
E(1 + V_i)/L_i & EV_i/RLC_i & (E/L_iC_i)^2(1/L_i + V_i) & -E(V_i^2)/L_iC_i(V_d + 1) \\
EV_d(1 + V_i)/RC_i - E/L_iC_i + EV_d/L_iC_i & (E/RLC_i)^2(1/L_i + V_i) & -(E(1/V_i^2) + 1/V_i)(1/L_i + V_i) & E(V_i)/RC_iV_i + E(V_i)/L_iC_i(V_d + 1) \\
E(V_d + 1)/L_i & -EV_d/RC_i & (E/V_i^2)/C_i(V_d + 1) & E(V_i)/C_i(V_d + 1) + (EV_d/L_iC_i)(1/L_i + V_i) \\
0 & EV_d/V_i^2C_i & (1/V_i^2) & E(V_i)/C_i(V_d + 1)
\end{bmatrix}
\]

\[\det(M) \neq 0\]  

(1.61)

Our model is observable too at the two states variables as is proven below:

For \( \bar{y} = \bar{x}_2 \), gives

\[
N = \begin{bmatrix}
0 & -1/L_i(V_d + 1) & 0 & -\frac{1}{L_i} + \frac{1}{L_i}V_i^2)/C_i(V_d + 1)^2 \\
1 & 0 & -(\frac{1}{L_i} + \frac{1}{L_i}V_i^2)/C_i(V_d + 1)^2 & 0 \\
0 & V_i / L_iC_i(V_d + 1) & 0 & -V_i(\frac{L_i}{C_i(V_d + 1)}(1/L_i + V_i^2)) - \frac{V_i}{L_iC_iC_i(V_d + 1)} \\
0 & 0 & -V_i / L_iC_i(V_d + 1) & \frac{V_i}{RL_iC_iC_i(V_d + 1)}
\end{bmatrix}
\]

\[\det(N) = 0\]  

(1.62)

For \( \bar{y} = \bar{x}_1 \), gives

\[
N = \begin{bmatrix}
0 & 0 & 0 & \frac{V_i}{L_iC_iC_i(V_d + 1)} \\
0 & 0 & -V_i / L_iC_iC_i(V_d + 1) & \frac{V_i}{RL_iC_iC_i(V_d + 1)} \\
0 & 1/C_2 & -1/RC_2 & -\frac{V_i^2}{E_iC_iC_i(V_d + 1)^2} - (1/C_2)^2(\frac{1}{L_i} - \frac{1}{RC_2}) \\
1 & -1/RC_2 & -(1/C_2)(\frac{1}{L_i} - \frac{1}{RC_2}) & \frac{1}{RL_iC_iC_i(V_d + 1)^2} + (1/RC_2)^2(\frac{1}{L_i} - \frac{1}{RC_2})
\end{bmatrix}
\]

\[\det(N) = 0\]  

(1.67)
1.7 Conclusion

The modeling has shown that the DC-DC converters are nonlinear systems (except for the Buck). The study in equilibrium state has established the converters static transfer functions, and permitted the understanding of the converters input-output relationships. Linearization around an equilibrium point has established the characteristics of local controllability and observability and thus prepared the ground for the application of linear control techniques.
Chapter 2

Linear State Feedback Control of DC-DC Converters

2.1. Introduction
In this chapter, we investigate the application of linear state feedback control techniques to improve the dynamic behavior of DC-DC converters. Sections 2.2, 2.3 and 2.4 present the necessary theoretical elements for the use of these techniques, namely, the state feedback by pole placement, the state feedback with integral action and the state feedback with observer. Section 2.5 presents the application of these techniques to different DC-DC converters modeled in the first chapter, and section 2.6 concludes the chapter. The underlying theory for this chapter is summarized from [4].

2.2. Poles placement
Considering the DC-DC converter linearized model:
\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]
(2.1)
If it is considered that the above system is completely controllable, then the control law is as follows:
\[
\dot{u} = -Kx
\]
(2.2)
Hence, the matrix $K$ of $(1 \times n)$ dimension is called the gain matrix.

Figure 2.1 shows the system defined by equation (2.1), open loop then closed loop mode, using the control law (2.2).

![Figure 2.1](image)

So the system becomes a closed loop control system as shown in Figure 2.1.b,
\[
\dot{x} = (A - BK)x
\]
(2.3)
Noting that the eigenvalues of the matrix $[A - BK]$ $(\mu_1, \mu_2, \ldots, \mu_n)$ are the desired closed loop poles.

In order to place the system poles via the state feedback, the system has to verify a necessary condition, which is the controllability, that is, the matrix
Chapter 2
Poles Placement Based State Feedback Control of DC-DC Converters

\[ M = \begin{bmatrix} B & AB & \ldots & A^{n-1}B \end{bmatrix} \] should be a full rank matrix.

In order to calculate the gain matrix \( K \), which is necessary for the poles placement, several ways are possible. Among them we cite:

### 2.2.1 Substitution

The gain \( K = \begin{bmatrix} k_1 & k_2 & \ldots & k_n \end{bmatrix} \) can be substituted directly into the characteristic polynomial of \( [sI - (A - BK)] \), equalizing this latter with the closed loop desired polynomial of converter model.

#### 2.2.2 Controllable form

The transformation matrix \( T = MW \), where \( W \) is defined by:

\[
W = \begin{bmatrix}
a_{n-1} & a_{n-2} & \ldots & a_1 & 1 \\
a_{n-2} & \ldots & a_1 & 1 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
a_1 & 1 & \ldots & 0 \\
1 & 0 & \ldots & 0 & 0
\end{bmatrix}
\] (2.4)

The gain matrix is then:

\[
K = \begin{bmatrix} \alpha_n - a_n & \alpha_{n-1} - a_{n-1} & \ldots & \alpha_1 - a_1 \end{bmatrix} T^{-1}
\] (2.5)

Where desired characteristic polynomial:

\[
(s - \mu_1)(s - \mu_2)\ldots(s - \mu_n) = s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_{n-1} s + \alpha_n
\] (2.6)

And the characteristic polynomial:

\[
|sI - A| = s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n
\] (2.7)

### 2.2.3 Ackermann formula

To get the gain matrix \( K \), it is possible using the Ackermann formula which is given as the following equation:

\[
K = \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} M^{-1} \Phi(A)
\] (2.8)

\[ \Phi(A) = A^n + \alpha_1 A^{n-1} + \ldots + \alpha_{n-1} A + \alpha_n I \] (2.9)
2.3. State feedback with integral action

To assess more realistic situations, we suppose that the converter is subject to static disturbance $d'$ due to load variations, that is:

$$\dot{x} = A\dot{x} + B\ddot{u}$$
$$\ddot{y} = C\dot{x} + d'$$

(2.10)

Then, the state feedback can't drive the output $\ddot{y}$ to zero. It is necessary to introduce an integral action in order to eliminate the disturbance effect. To this end, we introduce the additional state variables:

$$\ddot{x}_i = \int C\dot{x}$$

(2.11)

Or equivalently:

$$\ddot{x}_i = C\ddot{x}$$

(2.12)

Then, the augmented model is given by:

$$\dot{z} = A'z + B'\ddot{u}$$

(2.13)

With $z^T = [\dot{x} \ddot{x}_i]$ and

$$A' = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, B' = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

(2.14)

The new state feedback control law is given by:

$$\ddot{u} = -K^*z = -\begin{bmatrix} K & K_i \end{bmatrix} \begin{bmatrix} \dot{x} \\ \ddot{x}_i \end{bmatrix}$$

(2.15)

Which can be interpreted as a state feedback control law involving $(n + m)$ dimensional augmented state vector formed by the state vector $\dot{x}$ and the integrator state vector $\ddot{x}_i$.

The matrix $K^*$ of $(1 \times (n + m))$ dimensional

The closed loop system is given by:

$$z = [A' - B'K^*]z = \begin{bmatrix} A - BK & Bk_i \\ C & 0 \end{bmatrix}z$$

(2.16)

If the desired behavior is specified through the selection of $n + m$ desired eigenvalues $\mu_1, \mu_2, \ldots, \mu_{n+m}$, then, the gain vector $K^*$ can be computed using one the previously mentioned methods.

2.4. Observer based state feedback

2.4.1 Introduction

In the above part of this chapter, it is considered that all the state variables are available for the feedback. However, practically, some variables may not be available. therefore, the estimate phase of those variables is definitely an essential step, means, that we must estimate the non-measurable state variables so as to elaborate the control signals.
In case if the state observer estimates all the system state variables, even if some of those variables are accessible to direct measurement, this observer type is known as a full order state observer (fig. 2.2).

\[
\dot{x} = Ax + Bu \\
y = Cx
\]

A necessary condition to the realization of a state observer is that the converter linearized model is observable. This condition is realized if the observability matrix

\[
N = \begin{bmatrix} C^T & A^T C^T & \cdots & (A^T)^{n-1} C^T \end{bmatrix}
\]

has a \(\text{rank}(N) = n\).

The full order observer is defined by:

\[
\dot{\hat{x}} = A\hat{x} + Bu + K_e (\hat{y} - \hat{y}) \\
\dot{\hat{y}} = C\hat{x}
\]

The estimation (or observation) error \(e = \bar{x} - \hat{x}\) is defined by:

\[
\dot{e} = (A - K_eC)e
\]

Where \(K_e\) is the observer gain vector.

If \(K_e\) is chosen such as the matrix \((A - K_e C)\) is stable, then, the estimation error \(e(t) = e^{(A - K_e C)t}e(0)\)

converges exponentially to zero. That is, \(e(t) \to 0 \Rightarrow \bar{x} \to \hat{x}\).

If the desired observer behavior is specified through the selection of \(n\) desired eigenvalues \(\rho_1, \rho_2, \ldots, \rho_n\), then, the gain vector \(K_e\) can be computed using one the following methods.

**2.4.2 Substitution**

Supposing \(\hat{x}\) is a vector of the dimensional 3, under circumstances, the gain matrix is drawn out as following:

\[
K_e^T = \begin{bmatrix} k_{e1} & k_{e2} & \cdots & k_{en} \end{bmatrix}
\]
The matrix substitution $K_e$ into the desired characteristic polynomial is found to be given by:

$$[sI - (A - K_eC)] = (s - \mu_1)(s - \mu_2)(s - \mu_3)$$

(2.22)

By doing an equalization of the coefficients of the same power on $s$ of the two sides of the latter equation, then the values $k_{e1}, k_{e2}$ and $k_{e3}$ can be easily determined.

### 2.4.3 Observable form

The transformation matrix $T$ is described as:

$$T = (WN^T)^{-1}, \quad N \text{ is the matrix of the observability}$$

$$N = \begin{bmatrix} C^T & A^T C^T & \ldots & (A^T)^{n-1} C^T \end{bmatrix}$$

(2.23)

Where $W$ is defined by:

$$|sI - A| = s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n$$

(2.24)

And the desired characteristic equation for the error dynamic:

$$(s - \mu_1)(s - \mu_2)\ldots(s - \mu_n) = s^n + \alpha_1s^{n-1} + \alpha_2s^{n-2} + \ldots + \alpha_{n-1}s + \alpha_n = 0$$

(2.25)

So:

$$K_e = T^{-1}[\alpha_{n} - a_{n} \quad \alpha_{n-1} - a_{n-1} \quad \ldots \quad \alpha_{1} - a_{1}]$$

(2.26)

The equation (2.26) gives directly the gain matrix $K_e$ of our observer.

### 2.4.4 Ackermann formula

So as to calculate the observer gain, we can use the Ackermann formula of which its form is given as follows:

$$K_e = \phi(A)(N^T)^{-1} = \phi(A)$$

(2.27)

Where $\phi(s)$ is the desired characteristic polynomial of the observer:

$$\phi(s) = (s - \mu_1)(s - \mu_2)\ldots(s - \mu_n) = s^n + \alpha_1s^{n-1} + \alpha_2s^{n-2} + \ldots + \alpha_{n-1}s + \alpha_n$$

(2.28)

With $\mu_1, \mu_2, \ldots, \mu_n$ are the desired eigenvalues, whereas we have:

$$\phi(A) = A^n + \alpha_1A^{n-1} + \alpha_2A^{n-2} + \ldots + \alpha_{n-1}A + \alpha_nI_n$$

(2.29)
2.5. Application:

In this section, we’re about studying the application of the linear state feedback control on the DC to DC converters that have been analyzed within the first chapter, starting by the application of poles placement without and with integral action, then the application of the full order observer.

For each converter, the calculation steps are as follows:

1) Feedback

1. Fixing the desired capacitor voltage.
2. Calculating the equilibrium points as shown within chapter 1.
3. Linearizing the converter model at the desired operating points, to have

\[ \dot{x} = A\dot{x} + B\dot{u} \]
\[ \tilde{y} = C\tilde{x} \]

4. Fixing the specification of the closed loop response according to a specified overshoot or a settling time or both. Those specifications are defined by two parameters \( \omega_n \) and \( \zeta \), such that:

a) For the overshoot specification: \( M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \)

b) For the settling time specification we use: \( e^{-\zeta\omega_n t} \)

The damping ratio \( \zeta \) is usually taken as \( 0.7 \sim 2 \).

5. Calculating the gain matrices \( K \) and \( K_I \) using the follows Matlab command:

\[ K = \text{place}(A, B, P) \]

Where P is the vector of the desired eigenvalues

Then, the average control is given by:

\[ u_{av} = \overline{u_{av}} - \dot{x} - \overline{u_{av}} - K(x - \overline{x}) \]

In order to apply \( u_{av} \), it must be converted to gate signal \( u = [0, 1] \), this function is realized via an PWM technique as shown is the figure 2.3. The carrier signal has a triangular waveform with 40 kHz frequency. The control signal \( u_{av} \) is compared with the carrier signal, and the comparator output is switched at every intersection instant, so the gate signal \( u \) is of constant frequency and variable duty ratio (fig. 2.4).

In all simulations the MOSFET is taken as open circuit when it is off and it is equivalent to 0.7V voltage source in series with 50m\( \Omega \) resistance, when it is on. Similarly, the Diode is taken as open circuit when it is off and it is equivalent to 0.4V voltage source in series with 25m\( \Omega \) resistance, when it is on [1]. The converters structures and parameters values are taken from [2].
II) Observers

We suppose that only the voltages are measurable, and build an observer for estimating the currents.

Full-order observer form:
\[ \dot{\hat{x}} = A\hat{x} + B\hat{u}_w + K_c(\tilde{y} - \hat{y}) \]
\[ \hat{y} = C\hat{x} \]

The calculation of the gain matrix is as follows:

1. Choosing \( \omega_n \) and \( \zeta \).
2. Using the place Matlab function: \( K_e = \text{place}(A', C', P) \), \( P \) the desired poles.
2.5.1. Buck converter:

The assigned values for the buck converter are given by: \( E = 24V, L = 15.91mH, C = 50\mu F \) and \( R = 25\Omega \).

The desired average voltage is: \( \overline{V}_d = V_dE = 12V \), which yields \( \overline{V}_i = 0.4800 \) and \( \overline{V}_u = 0.5 \).

The linearized model is given as follows:

\[
\dot{x} = \begin{bmatrix} 0 & -62.854 \\ 20000 & -800 \end{bmatrix} x + \begin{bmatrix} 1508.5 \\ 0 \end{bmatrix} u_{av}
\]

This average model is controllable as proven within the first chapter.

![Fig. 2.6: Buck open loop response](image)

From the open loop characteristic, we get:

\( \omega_n = 1121.2\text{rad/sec}, \zeta = 0.35676 \).

It is concluded from the above curves that the response is slower and present 25% overshoot peak for the voltage and 62.5% for the current. Noting the time scale of the graphs, settling time in this graph is about 15 msec.

For better performance, the use of state feedback control will greatly reduce both the peak overshoots and deviations of the two \( i \) and \( v \), with acceptable settling time in the millisecond range.
a. State feedback

The average law control:

\[ \tilde{u}_{av} = -K\dot{x} = -k_1\dot{x}_1 - k_2\dot{x}_2 \]

The chosen parameters \( \zeta = 1, \omega_n = 2000 \text{ rad/sec} \)

The gain matrix of those parameters is:

\[ K = \begin{bmatrix} 2.1213 & 0.006064 \end{bmatrix} \]

After the application of feedback control we obviously notice the improvement on the buck converter performance, i.e. faster response, less deviations and low amplitude peaks. Even though this positive effect on the system response still has the inefficiency of the non-achieved zero static error that is approximated around 12.5% (fig. 2.8).

A zero static error dynamic controller for buck converter is designed to achieve zero voltage steady-state error, thus it is proposed adding an integrator to state feedback control that will serve eliminating non-zero error as will be demonstrated below.
b. State feedback with Integral action:

The augmented Buck converter model is:

\[
\dot{z} = \begin{bmatrix}
0 & -62.854 & 0 \\
20000 & -800 & 0 \\
0 & 1 & 0
\end{bmatrix} z + \begin{bmatrix} 1508.5 \end{bmatrix} \tilde{u}_{av}
\]

With

\[
z = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \int \dot{x}_2 \end{bmatrix}
\]

The augmented control law is given by:

\[
\tilde{u}_{av} = -k_1 \dot{x}_1 - k_2 \dot{x}_2 - k_i \int \dot{x}_2
\]

According to augmented Buck converter model, the desired polynomial is:

\[
(s + p)(s^2 + 2\zeta_\omega n s + \omega_n^2) = s^3 + (p + 2\zeta_\omega n) s^2 + (\omega_n^2 + 2p\zeta_\omega n) s + p\omega_n^2
\]

The chosen parameters: \( \zeta = 2, \omega_n = 1000 \text{rad/sec}, p = 400. \)

Then the augmented gain matrix is: \( K_f = [2.3865 \ -0.0509475 \ 13.2583] \)

---

Fig. 2.9. Buck state feedback + I control
Fig. 2.10: Buck closed loop response. (a) R=25, (b) resistance variation R=20.

The output voltage integral has been introduced as additional state variable as shown in Fig. 2.9, this integral action has successfully achieved a zero steady-state error that wasn’t be possible relying just on state feedback control. This approach has well proven its efficiency when having load variation, particularly, although the load is down to 20 ohms, the integrator is still effective on the converter response by providing an achieved zero static error.

c. Observer based state feedback control

The observability condition has been verified in the first chapter.

The chosen parameters for the observer: \( \zeta = 1, \omega_n = 4000 \text{rad/sec} \). The appropriate gain matrix for those parameters is given by:

\[
K_s = \begin{bmatrix} 737.1 \\ 7200 \end{bmatrix}
\]

The full order observer structure:

\[
\dot{x} = \begin{bmatrix} 0 & -800 \\ 20000 & -8000 \end{bmatrix} x + \begin{bmatrix} 1508.5 \\ 0 \end{bmatrix} \bar{u}_{av} + \begin{bmatrix} 737.1 \\ 7200 \end{bmatrix} \bar{x}_2
\]

The average law control is:

\[
\bar{u}_{av} = \bar{u}_{av} - k_1 \dot{x}_1 - k_2 \bar{x}_2
\]
The result gotten from the above graphs shows that the estimated current converges to the inductor current.

2.5.2 Boost converter

The assigned values for the boost converter are given by:

\[ E = 24V, L = 15.91mH, C = 50\mu F \] and \[ R = 52\Omega \]

The desired average voltage is: \[ \bar{V}_d = V_d E = 36V \], then \[ \bar{x}_i = 1.0385 \] and \[ \bar{u}_{av} = 0.3333 \]

The linearized model is given as follows:

\[
\begin{bmatrix}
  0 & -42 \\
  13333 & -385
\end{bmatrix}
\begin{bmatrix}
  \bar{x} \\
  \bar{u}_{av}
\end{bmatrix}
+ \begin{bmatrix}
  2263 \\
  -20769
\end{bmatrix}
\]
Fig. 2.13: Boost open loop response

From the open loop characteristic, we get:

\[ \omega_n = 2242.38 \text{ (rad / sec)} \] \quad \text{and} \quad \zeta = 0.1784. 

It is obviously shown, reading the slower response boost converter with about 30 msec settling time range. The response presents overshoot peaks of 25% for voltage and 188.8% for the current, which causes undesirable deviations before the steady-state.

For high efficiency, state feedback control will handle such mentioned problems by providing a faster response, and reducing the overshoots and deviations occurred within open loop application.

a. State feedback control:

The boost model is controllable as shown in the first chapter.

The average law control:

\[ \ddot{u}_{av} = -K\ddot{x} = -k_1\ddot{x}_1 - k_2\ddot{x}_2 \]

The chosen parameters \( \zeta = 1.8, \omega_n = 2000 \text{ rad / sec} \)

The gain matrix of those parameters is:

\[ K = \begin{bmatrix} 2.2809 & -0.0570004 \end{bmatrix} \]
With no question, state feedback has well enhanced the boost converter response giving an improved closed-loop behavior upon open-loop behavior, by making it faster, faint oscillations and a bit low overshoots. In addition the output voltage seems having a problem in achieving a reachable desired voltage due to steady-state error which is about 9.7%, therefore an integrator has to be added to state feedback for make sure providing a zero steady-state error.

b. State feedback with integral action:

The boost converter expanded model is:

\[
\dot{z} = \begin{bmatrix} 0 & -42 & 0 \\ 13333 & -385 & 0 \end{bmatrix} z + \begin{bmatrix} 2263 \\ -20769 \end{bmatrix} \tilde{u}_{av}
\]

With \( z = [\ddot{x}_1 \quad \ddot{x}_2 \quad \int \ddot{x}_2] \).
The augmented control law is given by:
\[ \tilde{u}_{av} = -k_1 \bar{x}_1 - k_2 \bar{x}_2 - k_3 \int \bar{x}_2 \]

According to augmented Boost converter model, the desired polynomial is:
\[ (s + p)(s^2 + 2\zeta \omega_n s + \omega_n^2) = s^3 + (p + 2\zeta \omega_n)s^2 + (\omega_n^2 + 2p\omega_n)s + p\omega_n^2 \]

The chosen parameters: \( \zeta = 1.8, \omega_n = 1000 rad/sec, p = 100. \)

Then the augmented gain matrix is:
\[ K_f = \begin{bmatrix} 0.2965 & -0.0122 & 4.1432 \end{bmatrix} \]

From the above graphs we can well see that the static error that was occurred within state feedback control is now completely eliminated, which means that the output voltage is exactly the desired one even with load variation. The desired voltage is now fully reachable, but this going to cause some overshoots and a bit slow settling time.
c. Observer based state feedback control

The boost model is defined as follows:

$$\dot{x} = \begin{bmatrix} 0 & -42 \\ 13333 & -385 \end{bmatrix} x + \begin{bmatrix} 2263 \\ -20769 \end{bmatrix} u_{av}$$

The observability condition has fulfilled within the first chapter.

The average law control:

$$\tilde{u}_{av} = -K\dot{x} = -k_1\dot{x}_1 - k_2\dot{x}_2$$

The chosen parameters for the observer: \( \zeta = 1.8, \omega_n = 4000\text{rad/sec} \)

The appropriate gain matrix for those parameters is given by:

$$K_e = \begin{bmatrix} 1158.1 \\ 13900 \end{bmatrix}$$

The full order observer structure:

$$\dot{\hat{x}} = \begin{bmatrix} 0 & -1200.1 \\ 13333 & -14285 \end{bmatrix} \dot{\hat{x}} + \begin{bmatrix} 2263 \\ -20769 \end{bmatrix} \tilde{u}_{av} + \begin{bmatrix} 1158.1 \\ 13900 \end{bmatrix} \dot{x}_2$$

![Observer](image)

Fig.2.18: Boost state feedback with full order observer.
Using the results of our graphs, we can compare the estimated current with the real current of the inductor of the yielded curve by approximating the estimation error that is 4.14% although the mentioned estimation error still has clear advantage estimating the real inductor current.

2.5.3 Buck-Boost converter

The assigned values for the buck-boost converter are given by:
\[ E = 25V, L = 15.91mH, C = 470\mu F \text{ and } R = 25\Omega \]

The desired average voltage is:
\[ \bar{x} = V_dE = -25V \text{ where } \bar{x} = 0.9615 \text{ and } \bar{u}_{av} = 0.5 \]

The linearized model is given as follows:
\[
\dot{\bar{x}} = \begin{bmatrix}
0 & -31.4 \\
-1063.8 & -40.9
\end{bmatrix} \bar{x} + \begin{bmatrix}
3142.7 \\
2045.8
\end{bmatrix} \bar{u}_{av}
\]

This model is controllable and observable as proven within the first chapter.
Within the open loop characteristic, we get: $\omega_n = 182.846 (rad/sec), \zeta = 0.1119$

The full state feedback control will assure enhancing the slow response buck-boost converter by improving the settling time (15msec), which means a faster response, and reducing the overshoots that are about 54% for voltage 330% for current.

a. State feedback control:

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -31.4 \\ -1063.8 & -40.9 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 3142.7 \\ 2045.8 \end{bmatrix} \bar{u}_{av}$$

The average law control:

$$\bar{u}_{av} = -K\bar{x} = -k_1\bar{x}_1 - k_2\bar{x}_2$$

The chosen parameters $\zeta = 1, \omega_n = 300 rad/sec$

The gain matrix of those parameters is:

$$K = \begin{bmatrix} 0.1821 \\ -0.0064 \end{bmatrix}$$
As mentioned before in buck and boost converters parts, state feedback has proven once more again its efficiency by improving the buck-boost response, this latter is now more faster with lower overshoots and less oscillations. Besides to these advantages still there is the disadvantage of static error that is about 2.4% which makes a problem to achieve the desired voltage. So for more improved performance, a state feedback with integrator added has been designed to work out such problem of non-zero static error.

b. State feedback with Integral action

The buck-boost converter error model is:

\[
\begin{bmatrix}
0 & -31.4 & 0 \\
-1063.8 & -40.9 & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\int \dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
3142.7 \\
2045.8 \\
0
\end{bmatrix}
= \begin{bmatrix}
\dot{u}_{av}
\end{bmatrix}
\]

With \( z = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \int \dot{x}_2 \end{bmatrix} \).

The augmented control law is given by:

\[
\ddot{u}_{av} = -k_1 \dot{x}_1 - k_2 \dot{x}_2 - k_i \int \dot{x}_2
\]

According to augmented Buck-boost converter model, the desired polynomial is:

\[
(s + p)(s^2 + 2\zeta \omega_n s + \omega_n^2) = s^3 + (p + 2\zeta \omega_n) s^2 + (\omega_n^2 + 2p\zeta \omega_n) s + p\omega_n^2
\]

The chosen parameters: \( \zeta = 1, \omega_n = 200 \text{rad/sec}, p = 400 \).

Then the augmented gain matrix is: \( K_I = \begin{bmatrix} 0.2659 & -0.0374 & -4.7857 \end{bmatrix} \)
As shown above, the desired voltage buck-boost converter has been perfectly reached, despite the load variation, the steady-state error is a proven zero. Still, there are noticeable overshoots and slower settling time than the one whose from state feedback control.

c. Observer-based state feedback control

The buck-boost model is defined as follows:

$$\dot{x} = \begin{bmatrix} 0 & -31.4 \\ -1063.8 & -40.9 \end{bmatrix} x + \begin{bmatrix} 3142.7 \\ 2045.8 \end{bmatrix} u_{av}$$

The observability condition has fulfilled within the first chapter.

The average law control:

$$\tilde{u}_{av} = -K\dot{x} = -k_1\dot{x}_1 - k_2\dot{x}_2$$

The chosen parameters for the observer: $\zeta = 1$, $\omega_n = 1000 \text{rad} / \text{sec}$

The appropriate gain matrix for those parameters is given by:
The full order observer structure:

\[
\dot{\hat{x}} = \begin{bmatrix} 0 & 940 \\ -1063.8 & -2000 \end{bmatrix} \dot{\hat{x}} + \begin{bmatrix} 3142.7 \\ 2045.8 \end{bmatrix} \bar{u}_{av} + \begin{bmatrix} -908.6 \\ 1959.1 \end{bmatrix} \hat{x}_2
\]

Fig. 2.25 Buck Boost state feedback with full order observer.

From the above graphs, it’s obviously noticed that the estimation error has successfully converged to zero, means that \( \hat{i} \simeq i \)

2.5.4. Cuk converter

The assigned values for the Cuk converter are given by:

\[ E = 100V, L_1 = L_2 = 30mH, C_1 = 150\mu F, C_2 = 50\mu F \] and \( R = 10\Omega \)
The desired average voltage is: $\bar{x}_4 = V_L E = -150V$, then $\bar{x}_4 = 22.5, \bar{x}_2 = 250, \bar{x}_1 = -15$ and $\bar{u}_{av} = 0.6$

The linearized model is given as follows:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & -13.3333 & 0 & 0 \\ 2666.7 & 0 & 4000 & 0 \\ 0 & -20 & 0 & -33.3333 \\ 0 & 0 & 20000 & -2000 \end{bmatrix} \bar{x} + \begin{bmatrix} 8333.3 \\ -100000 \\ -8333.3 \\ 0 \end{bmatrix} \bar{u}_{av}$$

This model is controllable and observable as proven within the first chapter.

From the figure 2.27 we notice, that in comparison with the desired tensions and currents values, the steady state errors are very important. Moreover, the responses present very large oscillations and rather slow times of establishment.

By applying state feedback command it is possible getting a faster response, less oscillations and a reduced overshoots.

**a. state feedback control:**

The average law control:

$$\bar{u}_{av} = -K\bar{x} = -k_1\bar{x}_1 - k_2\bar{x}_2 - k_3\bar{x}_3 - k_4\bar{x}_4$$

According to Cuk converter model, the desired polynomial is:

$$\left(s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2\right)\left(s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2\right)$$

The chosen parameters $\zeta_1 = \zeta_2 = 1, \omega_{n1} = 500rad / sec, \omega_{n2} = 1000rad / sec$

The gain matrix of those parameters is:
The closed-loop response Cuk converter has been effectively enhanced from a slow response to a faster one and from high oscillated overshoots to less oscillations and low peaks. Although state feedback doesn’t provide an achieved zero steady-state error, but in case adding an integrator to state feedback this error will be completely removed.
b. State feedback with integral action

The Cuk converter error model is:

\[
\begin{bmatrix}
0 & -13.3333 & 0 & 0 & 0 \\
2666.7 & 0 & 4000 & 0 & 0 \\
0 & -20 & 0 & -33.3333 & 0 \\
0 & 0 & 20000 & -2000 & 0 \\
0 & 0 & 0 & 2 & 0
\end{bmatrix}
\begin{bmatrix}
z \\\nz \\\nz \\\nz \\\n\dot{z}
\end{bmatrix}
+ \begin{bmatrix}
8333.3 \\
-100000 \\
-8333.3 \\
0
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_w
\end{bmatrix}
\]

With 
\[
z = \begin{bmatrix}
\ddot{x}_1 \\
\ddot{x}_2 \\
\ddot{x}_3 \\
\ddot{x}_4 \\
\int \ddot{x}_4
\end{bmatrix}
\]

The augmented control law is given by:

\[
\tilde{u}_w = -k_1 \ddot{x}_1 - k_2 \ddot{x}_2 - k_3 \ddot{x}_3 - k_4 \ddot{x}_4 - k_5 \int \ddot{x}_4
\]

According to augmented Cuk converter model, the desired polynomial is:

\[
(s + p) \left( s^2 + 2\zeta_1 \omega_n s + \omega_n^2 \right) \left( s^2 + 2\zeta_2 \omega_n s + \omega_n^2 \right)
\]

The chosen parameters:
\[
\zeta_1 = \zeta_2 = 1, \quad \omega_n = 500 \text{ rad/sec}, \quad \omega_n = 500 \text{ rad/sec}, \quad p = 40 \text{ rad/sec}
\]

Then the augmented gain matrix is:

\[
K_I = \begin{bmatrix}
0.0253 & -0.0011 & 0.0339 & -0.0036 & -0.1688
\end{bmatrix}
\]

Fig. 2.30: Cuk state feedback + I control
The output voltage integral is treated as additional state feedback for reason to remove the static error, so from the Cuk response it can be easily seen that this error has ended up to zero, even though load variation both desired voltages are now greatly reachable. But this improvement will induce a slower settling time if it is compared with the one gotten from state feedback control.

c. Observer based state feedback control

The Cuk model is defined as follows:

\[
\dot{x} = \begin{bmatrix} 0 & -13.3333 & 0 & 0 \\ 2666.7 & 0 & 4000 & 0 \\ 0 & -20 & 0 & -33.3333 \\ 0 & 0 & 20000 & -2000 \end{bmatrix} x + \begin{bmatrix} 8333.3 \\ -10000 \\ -8333.3 \\ 0 \end{bmatrix} u_{av}
\]

The observability condition has fulfilled within the first chapter

The average law control:

\[
\tilde{u}_{av} = -K\tilde{x} = -k_1\hat{x}_1 - k_2\hat{x}_2 - k_3\hat{x}_3 - k_4\hat{x}_4
\]

The chosen parameters for the observer:

\[
\zeta_1 = \zeta_2 = 1, \omega_{n1} = 500 \text{ rad/ sec}, \omega_{n2} = 1000 \text{ rad/ sec}
\]

The appropriate gain matrix for those parameters is given by:

\[
K_e = \begin{bmatrix} -129.9 \\ -2883.3 \\ 123.4 \\ 1000 \end{bmatrix}
\]
The full order observer structure:

\[
\begin{bmatrix}
0 & -13 & 0 & 130 \\
2667 & 0 & 4000 & 2883 \\
0 & -20 & 0 & -157 \\
0 & 0 & 20000 & -3000
\end{bmatrix}
\begin{bmatrix}
v \end{bmatrix}
\begin{bmatrix}
x \\
x \end{bmatrix} + \begin{bmatrix}
8333.3 \\
-100000 \\
-8333.3 \\
0
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_{av} \\
\tilde{u}_{av}
\end{bmatrix} + \begin{bmatrix}
-129.9 \\
-2883.3 \\
123.4 \\
1000
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_4
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} = \begin{bmatrix}
v \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\tilde{u}_{av} \\
\tilde{u}_{av}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\dot{\tilde{x}}_1 \\
\dot{\tilde{x}}_2 \\
\dot{\tilde{x}}_3
\end{bmatrix} = \begin{bmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_1 \\
\tilde{x}_2 \\
\tilde{x}_3
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
\tilde{v}_1 \\
\tilde{v}_2
\end{bmatrix}
\]

Observer

Fig. 2.32. Cuk state feedback with full order observer.

Fig. 2.33: Cuk closed loop response with full order observer.

From the above graphs, it’s obviously noticed that the estimation error has successfully converged to zero.
2.5.5. Zeta converter:

The assigned values for the Zeta converter are given by:

\[ E = 100V, L_1 = 600mH, L_2 = 10mH, C_1 = C_2 = 10\mu F \] and \( R = 40\Omega \)

The desired average voltage is: \( \bar{x}_4 = V_2E = 200V \), then \( \bar{x}_1 = 10, \bar{x}_2 = 200, \bar{x}_3 = 5 \) and \( \bar{u}_{av} = 0.666 \)

The linearized model is given as follows:

\[
\dot{x} = \begin{bmatrix} 0 & -555.5556 & 0 & 0 \\ 33333 & 0 & -66667 & 0 \\ 0 & 66.6667 & 0 & -100 \\ 0 & 0 & 100000 & -2500 \end{bmatrix} x + \begin{bmatrix} 500000 \\ -150000 \\ 30000 \\ 0 \end{bmatrix} \bar{u}_{av}
\]

This model is controllable and observable as proven within the first chapter.

From the open loop response Zeta converter we can well notice that in comparison with the desired voltages and currents values, the static errors are very important. Moreover, the responses present very large oscillations and rather slow settling times.

The application of state feedback control will definitely improve the Zeta performance by providing faster response, less deviations and low overshoots.
a. State feedback control

\[
\begin{bmatrix}
    0 & -555.5556 & 0 & 0 \\
    33333 & 0 & -66667 & 0 \\
    0 & 66.6667 & 0 & -100 \\
    0 & 0 & 100000 & -2500
\end{bmatrix}
\begin{bmatrix}
    \dot{x} \\
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3
\end{bmatrix}
+
\begin{bmatrix}
    500000 \\
    -1500000 \\
    30000 \\
    0
\end{bmatrix}
\tilde{u}_{av}
\]

The average law control:

\[
\tilde{u}_{av} = -K\dot{x} = -k_1\dot{x}_1 - k_2\dot{x}_2 - k_3\dot{x}_3 - k_4\dot{x}_4
\]

According to Zeta converter model, the desired polynomial is:

\[
(s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2)(s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2) = s^4 + 2(\zeta_1\omega_{n1} + \zeta_2\omega_{n2})s^3 + (\omega_{n2}^2 + \omega_{n1}^2 + 4\zeta_1\zeta_2\omega_{n1}\omega_{n2})s^2 + 2\omega_{n1}\omega_{n2}(\zeta_1\omega_{n2} + \zeta_2\omega_{n1})s + (\omega_{n1}\omega_{n2})^2
\]

The chosen parameters \(\zeta_1 = \zeta_2 = 0.9, \omega_{n1} = \omega_{n2} = 3000\,\text{rad/\text{sec}}\)

The gain matrix of those parameters is:

\[
K = \begin{bmatrix}
0.0136 & 0.0007 & 0.0142 & -0.0003
\end{bmatrix}
\]

![Fig.2.35: Zeta state feedback control](image-url)
The application of feedback command greatly improved the Zeta converter performance, means faster response, less deviations and low overshoots, despite this improvement, it is still affected by the inefficiency of the steady-state error, a zero static error dynamic controller for Zeta converter is designed to achieve zero voltage steady-state error, thus an integrator will be added to state feedback control that will effectively eliminate the non-zero error as will be shown next.

b. State feedback with Integral action:

The Zeta converter error model is:

\[
\begin{bmatrix}
0 & -555.5556 & 0 & 0 & 0 \\
33333 & 0 & -66667 & 0 & 0 \\
0 & 66.6667 & 0 & -100 & 0 \\
0 & 0 & 100000 & -2500100 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
z_1 \\
z_2 \\
z_3 \\
z_4 \\
z_5
\end{bmatrix}
= \begin{bmatrix}
500000 \\
-1500000 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_{nv}
\end{bmatrix}
\]

with

\[
z = [\tilde{x}_1 \, \tilde{x}_2 \, \tilde{x}_3 \, \tilde{x}_4 \, \int \tilde{x}_4]
\]

The augmented control law is given by:

\[
\tilde{u}_{nv} = -k_1 \tilde{x}_1 - k_2 \tilde{x}_2 - k_3 \tilde{x}_3 - k_4 \tilde{x}_4 - k_5 \int \tilde{x}_4
\]
According to augmented Buck converter model, the desired polynomial is:

\[(s + p)(s^2 + 2\zeta_1\omega_n \omega_1 s + \omega_1^2)(s^2 + 2\zeta_2\omega_n \omega_2 s + \omega_2^2)\]

The chosen parameters: \(\zeta_1 = \zeta_2 = 1, \omega_n = \omega_n = 3000 \text{rad/sec}, p = 500 \text{rad/sec}\)

Then the augmented gain matrix is: 

\[K_I = \begin{bmatrix} 0.0152 & 0.0012 & 0.0173 & -0.0005 & 0.243 \end{bmatrix}\]

As shown, the Zeta performance has well improved after the application of the same control approach but with integral action. With or without load variation, a zero steady-state error is actually obtained with an acceptable settling time and less oscillation have proven too.
c. Observer based state feedback control

The Zeta model is defined as follows:

\[
\dot{\tilde{x}} = \begin{bmatrix}
0 & -555.5556 & 0 & 0 \\
33333 & 0 & -66667 & 0 \\
0 & 66.6667 & 0 & -100 \\
0 & 0 & 100000 & -2500 \\
\end{bmatrix}
\begin{bmatrix}
\tilde{x} \\
\tilde{\dot{x}} \\
\tilde{\ddot{x}} \\
\tilde{\dddot{x}} \\
\end{bmatrix}
+ \begin{bmatrix}
500000 \\
-1500000 \\
30000 \\
0 \\
\end{bmatrix}
\tilde{u}_{av}
\]

The observability condition has fulfilled within the first chapter

The average law control:

\[
\tilde{u}_{av} = -K\ddot{x} = -k_1\dot{\tilde{x}}_1 - k_2\dot{\tilde{x}}_2 - k_3\dot{\tilde{x}}_3 - k_4\dot{\tilde{x}}_4
\]

The chosen parameters for the observer: \(\zeta_1 = \zeta_2 = 1, \omega_n_1 = \omega_n_2 = 8000rad/sec\)

The appropriate gain matrix for those parameters is given by:

\[
K_\varepsilon = \begin{bmatrix}
-11650 \\
196980 \\
3510 \\
29500 \\
\end{bmatrix}
\]

The full order observer structure:

\[
\dot{\tilde{x}} = \begin{bmatrix}
0 & -560 & 0 & 11650 \\
33333 & 0 & -66670 & -196980 \\
0 & 70 & 0 & -3610 \\
0 & 0 & 100000 & -32000 \\
\end{bmatrix}
\begin{bmatrix}
\tilde{x} \\
\tilde{\dot{x}} \\
\tilde{\ddot{x}} \\
\tilde{\dddot{x}} \\
\end{bmatrix}
+ \begin{bmatrix}
500000 \\
-1500000 \\
30000 \\
0 \\
\end{bmatrix}
\tilde{u}_{av}
+ \begin{bmatrix}
-11650 \\
196980 \\
3510 \\
29500 \\
\end{bmatrix}
\tilde{x}_4
\]

Fig. 2.39. Zeta state feedback with full order observer.
Fig. 2.40: Zeta closed loop response with full order observer.

From the above graphs, it’s obviously noticed that the estimation error has successfully converged to zero.
2.6. Conclusion:

This chapter has demonstrated the process of applying modern control design methods to regulator design for DC to DC converters. Full state feedback control for pole placement was applied – first without integral action, then with an integrator added, underlining the clear advantages of state feedback, that has a positive effect on response settling time, reducing the undesirable peak overshoots and serve having a less oscillated performance, referring that this approach doesn’t provide a zero static error, this latter has been solved by adding an integral action to state feedback control, that has proven its efficiency working out the steady-state error then the use of state estimation technique with Full-Order Estimator was discussed and simulated, that comes out with the advantages of the observer in estimating those non measurable state variables basically the current(s).
Chapter 3

Passivity based control of DC-DC converters

3.1 Introduction
Within this chapter, the behavior of DC-DC converters is being controlled this time with another control approach so-called based passivity control. In this part, we’ll try pursuing the application of passivity based control technique to improve the dynamic behavior of DC-DC converters. Sections 3.2, 3.3 and 3.4 present the theoretical elements necessary for the use of these techniques, namely, the based passivity and the based passivity control with non-linear observer. Section 3.5 presents the application of these techniques to different DC-DC converters modeled in the first chapter, and section 3.6 concludes the chapter. The PBC theory is taken from [2] and [3].

3.2 DC-DC Converters energetic models
The modeled DC-DC converters show a clear “Energy Management” structure. It is considered in this section that the general model of the static converters is given by:

\[ \dot{x} = J(u_{av})x - Rx + Bu_{av} + \varepsilon \]

Where \( x \in R^n \), \( u \in R^m \), \( B \in R^{nxm} \) matrix, \( D \) is a symmetric definite positive matrix, \( J(u_{av}) \) is an anti-symmetric matrix \( (J(u_{av}) + J^T(u_{av}) = 0) \), it is also a function of the variable \( u_{av} \):

\[ J(u_{av}) = J_0 + \sum_{i=1}^{m} J_i u_{av} \]

The matrix \( R \) is a symmetric semi-definite positive, i.e. \( R = R^T \geq 0 \). The term \( \varepsilon \in R^n \) represents the constant output voltage sources.

The term \( J(u_{av})x \) represents the conservative (workless) forces, and the term \( Rx \) represents the dissipative forces in the converter. The term \( Bu_{av} \) represents the acquisition of the energy.

An explanation of this terminology is that, if \( V = \frac{1}{2} x^T D x \) is the "energy state", so, its derivative with respect to time

\[ \dot{V} = x^T J(u_{av})x - x^T Rx + x^T Bu_{av} + x^T \varepsilon \]

exhibits a zero term \( x^T J(u_{av})x \), i.e., \( J(u_{av})x \) provides invariant part of the energy, a semi-definite negative term \( -x^T Rx \), and the term \( x^T Bu_{av} \) responsible of the acquisition or the subtraction of the energy of the converter. Whereas the passive output is defined as \( y = B^T x \), the latter term is simply the product \( yu_{av} \) so-called the acquisition rate.

3.3 Control design
For the desired equilibrium point \((x, u_{av})\), and using the fact that the matrix \( J(u_{av}) \) is affine in the variable \( u_{av} \), yields the follows relation:
\[ J(u_{av}) = J(\bar{x}_{av}) + \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} (u_{av} - \bar{x}_{av}) \]  

(3.3)

We get at equilibrium:

\[ 0 = J(\bar{x}_{av})\bar{x} - R\bar{x} + B\bar{x} + \varepsilon \]  

(3.4)

Defining the errors as next: \( e = x - \bar{x} \) and \( e_u = u_{av} - \bar{x}_{av} \) obviously \( \dot{e} = \dot{x} \).

The subtraction of (3.1) and (3.4), we get the following error dynamic:

\[ D\dot{e} = J(u_{av})e - Re + \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + Be_u \]  

(3.5)

Then we simply get:

\[ D\dot{e} = J(u_{av})e - R.e + \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + Be_u \]  

(3.6)

If the output error is defined as follows: \( e_y = \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + B \) \( e \), which is the passive output of the system (3.6), it can be written:

\[ D\dot{e} = J(u_{av})e - R.e + \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + Be_u \]  

(3.7)

\[ e_y = \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + B \] \( e \)  

(3.8)

In order to stabilize the error dynamic, the control law is chosen as follows:

\[ e_u = -\Gamma e_y = -\Gamma \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + B \] \( e \)  

(3.9)

Where \( \Gamma \) > 0 of dimension \((m \times m)\).

The closed-loop error dynamic is then as following:

\[ D\dot{e} = J(u_{av})e - R.e + \left[ \left. \frac{dJ(u_{av})}{du_{av}} \right|_{u_{av} = \bar{x}_{av}} \right] \bar{x} + Be_u \]  

(3.10)

For demonstrating the stability of the error dynamic (3.10), we evaluate the total derivation of the energy function \( V(e) = \frac{1}{2} e^T De \) that makes:
\[ \dot{V} = e^T J(u_{av})e - e^T \left( R + \left[ \frac{dJ(u_{av})}{du_{av}} \right] \bar{x} + B \right) \left[ \frac{dJ(u_{av})}{du_{av}} \right]^T \bar{x} + B \] 

(3.11)

\[ = -e^T \left( R + \left[ \frac{dJ(u_{av})}{du_{av}} \right] \bar{x} + B \right) \left[ \frac{dJ(u_{av})}{du_{av}} \right]^T \bar{x} + B \] \leq 0

The derivative \( \dot{V} \) is negative definite just if the following condition holds

\[ \left( R + \left[ \frac{dJ(u_{av})}{du_{av}} \right] \bar{x} + B \right) \left[ \frac{dJ(u_{av})}{du_{av}} \right]^T \bar{x} + B \geq 0 \] 

(3.12)

That implies the following control law:

\[ u_{av} = \bar{u}_{av} + \Gamma e_y = \bar{u}_{av} - \Gamma \left[ \frac{dJ(u_{av})}{du_{av}} \right]^T \bar{x} + B \] \( e \) 

(3.13)

makes the origin globally asymptotically stable, if the condition (3.12) called “dissipativity matching” is verified.

### 3.4 Non-linear observer

We consider a non-linear observer structure for the estimation of the non measurable states. Far from the proposed linear observer within the second chapter, the non-linear observer uses directly the non-linear model of the converter, which means that there is no need for the linearizing step.

Here we consider the converter model given by:

\[ D\dot{x} = J(u_{av})x - Rx + Bu_{av} + \varepsilon \]

(3.14)

\[ y = C^T x \]

Where \( x \in R^n, u_{av} \in R^m \) and \( y \in R^p \).

Since the converter is observable, the non-linear observer is given by:

\[ D\dot{\hat{x}} = J(u_{av})\hat{x} - R\hat{x} + Bu_{av} + \varepsilon + K(y - \hat{y}) \]

(3.15)

\[ \hat{y} = C^T \hat{x} \]

Then the estimation error is as follows:

\[ D\dot{\hat{e}} = J(u_{av})\hat{e} - R\hat{e} - K\hat{e}_y \]

(3.16)

\[ \hat{e}_y = C^T \hat{e} \]

Where \( \hat{e} = x - \hat{x} \) is the state estimation error, \( \hat{e}_y = y - \hat{y} \) is the output estimation error.

If the proposed gain is of the form: \( K = CL \), with the matrix \( L \in R^{p \times p} \) is positive definite, the error dynamic is then:

\[ D\dot{\hat{e}} = J(u_{av})\hat{e} - [R + CLC^T]\hat{e} \]

(3.17)

\[ \hat{e}_y = C^T \hat{e} \]
To demonstrate the stability of the estimation error dynamic (3.17), we evaluate the total derivation of the Lyapunov function 

\[ V(\dot{e}) = \frac{1}{2} \dot{e}^T D \dot{e} \]

that makes:

\[
\dot{V} = e^T J(u_{av}) \dot{e} - e^T [R + CLC^T] \dot{e} \\
= -\dot{e}^T [R + CLC^T] \dot{e} \leq 0
\]  

(3.18)

The derivative \( \dot{V} \) is negative definite just if the matrix \([R + CLC^T]\) is definite positive. This condition is called "dual dissipativity matching".

**3.5 Application**

**a. Passivity based control**

1. Determine the converter energetic average model. This model usually takes the form (3.1).
2. Define the constant desired operating values \((x, u_{av})\) (current, voltage).
3. Choose \(\Gamma\) in the control law (3.9).
4. Verify the dissipativity matching condition (3.12).
5. In order to apply \(u_{av}\), it must be converted to gate signal \(u = [0, 1]\), this function is realized via an PWM technique as shown within chapter 2.

**b. Non-linear observer**

1. Choose the observer gain \(K = CL\).
2. Verify the dual dissipativity matching condition.
3. Implement the observer as in (3.15).


Chapter 3  Passivity based control of DC-DC converters

3.5.1 BUCK CONVERTER

a. Passivity based control

The Buck average model:

\[ L\dot{x}_1 = -x_2 + u_{av}E \]
\[ C\dot{x}_2 = x_1 - \frac{1}{R} x_2 \]

This model can be written as follows:

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix} \dot{x} = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix} x - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix} x + \begin{bmatrix}
E \\
0
\end{bmatrix} u_{av}
\]

The desired voltage \( \bar{x}_2 = EV_a \) then \( \bar{x}_1 = \frac{E}{R} V_d \), and \( \bar{u}_{av} = V_d \).

The error dynamic: \( e_1 = x_1 - \bar{x}_1 \) \( e_2 = x_2 - \bar{x}_2 \) is given by:

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix} \dot{e} = \begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix} e - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix} e + \begin{bmatrix}
E \\
0
\end{bmatrix} e_a
\]

\[ e_y = \begin{bmatrix} E & 0 \end{bmatrix} e \]

The control law:

\[ u_{av} = \bar{u}_{av} + e = \bar{u}_{av} - \gamma e_y = V_d - \gamma \begin{bmatrix} E & 0 \end{bmatrix} e = V_d - \gamma E e_1 \]

Since the matrix:

\[
\begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix} + \gamma \begin{bmatrix}
E & 0 \\
0 & \frac{1}{R}
\end{bmatrix} = \begin{bmatrix}
\gamma E^2 & 0 \\
0 & \frac{1}{R}
\end{bmatrix} > 0
\]

The dissipativity matching condition is verified.

The PBC scheme is given in figure 3.3.

Figure 3.3: Buck passivity based control.
From the buck open loop response, the buck performance was slower, sensitive and had followed by undesirable deviations, but after the application of the passivity based control, the performance we get has a faster response, less deviations, low amplitude start-up peaks and almost achieved zero steady-state error, i.e. the buck behavior has been perfectly improved.

b. Non-linear observer

The non-linear observer form:

\[
\begin{bmatrix}
L & 0
0 & C
\end{bmatrix}
\dot{\hat{x}} =
\begin{bmatrix}
0 & -1
1 & 0
\end{bmatrix}
\hat{x} -
\begin{bmatrix}
0 & 0
0 & \frac{1}{R}
\end{bmatrix}
\dot{\hat{x}} +
\begin{bmatrix}
u_{av}E
k_1
k_2
\end{bmatrix}(x_2 - \hat{x}_2)
\]

The dynamic of the estimation error is given by:

\[
\dot{\hat{e}}^T = \begin{bmatrix}
\hat{e}_1 & \hat{e}_2
\end{bmatrix}, \text{ with } \hat{e}_1 = x_1 - \hat{x}_1, \hat{e}_2 = x_2 - \hat{x}_2
\]

\[
\begin{bmatrix}
L & 0
0 & C
\end{bmatrix}\dot{\hat{e}} =
\begin{bmatrix}
0 & -1
1 & 0
\end{bmatrix}\dot{\hat{e}} -
\begin{bmatrix}
0 & 0
0 & \frac{1}{R}
\end{bmatrix}\dot{\hat{e}} -
\begin{bmatrix}
k_1
k_2
\end{bmatrix}\hat{e}_2
\]

Considering the Lyapunov function; \( V = \frac{1}{2}(\dot{\hat{e}}_1^2 + C\dot{\hat{e}}_2^2) \) Then \( \dot{V} = k_1\hat{e}_1\dot{\hat{e}}_2 - (k_2 + \frac{1}{R})\hat{e}_2^2 \).

If \( k_1 = 0 \) and \( k_2 \geq 0 \) we get \( -(k_2 + \frac{1}{R})\hat{e}_2^2 \leq 0 \), that is negative semi-definite.

The observer:

\[
L\dot{\hat{x}}_1 = -\dot{x}_2 + Euv_{av}
\]

\[
C\dot{\hat{x}}_2 = \dot{\hat{x}}_1 - \frac{1}{R}\dot{x}_2 + k_2(x_2 - \hat{x}_2)
\]
Fig. 3.5: Buck non-linear observer.

![Figure 3.5: Buck non-linear observer.]

Fig. 3.6: Buck closed loop response with non-linear observer $\gamma = 0.02, k_2 = 0.01$.

It is clear that the observer performance is not good enough. There is a residual error on the estimation of the current, which induces an error on the tracking of the reference voltage.

3.5.2 Boost converter

a. Passivity based control

The Boost average model:

$$L\dot{x}_1 = -(1 - u_{ac})x_2 + E$$

$$C\dot{x}_2 = (1 - u_{ac})x_1 - \frac{1}{R}x_2$$

This model can be written as follows:
The desired voltage \( \bar{x}_2 = E V_d \), then \( \bar{x}_1 = \frac{E}{R} V_d^2 \) and \( \bar{u}_{av} = 1 - \frac{1}{V_d} \).

The error dynamic can be written as next
\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{e} = \begin{bmatrix} 0 & -(1 - u_{av}) \\ (1 - u_{av}) & 0 \end{bmatrix} e - \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} e + \begin{bmatrix} V_d E \\ -\frac{E}{R} V_d^2 \end{bmatrix} e_u
\]
\[
e_y = [V_d E - \frac{E}{R} V_d^2] e
\]

The control law:
\[
e_u = -\gamma e_y = -\gamma [V_d E - \frac{E}{R} V_d^2] e = -\gamma V_d E e_1 + \gamma \frac{E}{R} V_d^2 e_2
\]

The closed loop dynamic is then as follows:
\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{e} = \begin{bmatrix} 0 & -(1 - u_{av}) \\ (1 - u_{av}) & 0 \end{bmatrix} e - \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} e + \begin{bmatrix} V_d E \\ -\frac{E}{R} V_d^2 \end{bmatrix} [V_d E - \frac{E}{R} V_d^2] e
\]

Since the matrix:
\[
\begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{R} \end{bmatrix} + \gamma \begin{bmatrix} V_d E \\ -\frac{E}{R} V_d^2 \end{bmatrix} [V_d E - \frac{E}{R} V_d^2] = \begin{bmatrix} \gamma V_d^2 E^2 & -\gamma \frac{E^2}{R} V_d^3 \\ -\gamma \frac{E^2}{R} V_d^3 & 1 + \gamma \frac{E^2}{R^2} V_d^4 \end{bmatrix} > 0
\]

The dissipativity matching condition is verified.

The control law will be then as follows:
\[
u_{av} = \bar{u}_{av} - \gamma e_y = \bar{u}_{av} - \gamma V_d E e_1 + \gamma \frac{E}{R} V_d^2 e_2
\]

With \( e_1 = x_1 - \frac{E}{R} V_d^2 \) \( e_2 = x_2 - EV_d \)

![Figure 3.7: Boost passivity based control.](image)
Figure 3.8: Boost closed loop response $\gamma = 0.05$.

It is obviously had been shown reading the open loop slower response of boost converter with large settling time range, the response was also sensitive and was followed by start-up peak overruns and noticeable oscillations before the steady-state. Now the previous drawbacks are greatly reduced, means the boost closed-loop behavior has became faster, with reduced start-up peaks overshoots and no occurred oscillations, and last a nearly achieved zero static error. Otherwise the passivity based control has given absolutely an improved closed loop performance than the one we get from open loop characteristic.

b. Non-linear observer

The non-linear observer form

$$
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\hat{x} =
\begin{bmatrix}
0 & -(1 - u_{av}) \\
(1 - u_{av}) & 0
\end{bmatrix}
\hat{x} -
\begin{bmatrix}
0 & 0 \\
0 & 1/R
\end{bmatrix}
\hat{x} +
\begin{bmatrix}
k_1 \\
k_2
\end{bmatrix}
(x_2 - \hat{x}_2)
$$

The dynamic of the estimation error is given by:

$$
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\hat{e} =
\begin{bmatrix}
0 & -(1 - u_{av}) \\
(1 - u_{av}) & 0
\end{bmatrix}
\hat{e} -
\begin{bmatrix}
0 & 0 \\
0 & 1/R
\end{bmatrix}
\hat{e} -
\begin{bmatrix}
k_1 \\
k_2
\end{bmatrix}
e_2
$$

with $\hat{e}^T = [\hat{e}_1 \ \hat{e}_2]$, $\hat{e}_1 = x_1 - \hat{x}_1$, $\hat{e}_2 = x_2 - \hat{x}_2$.

Considering the Laypunov function $V = \frac{1}{2}(Le_1^2 + C\hat{e}_2^2)$, then $\dot{V} = -k_1\hat{e}_1\hat{e}_2 - (k_2 + \frac{1}{R})\hat{e}_2^2$.

For $k_1 = 0$ and $k_2 > 0$, we get $\dot{V} = -(k_2 + \frac{1}{R})\hat{e}_2^2 \leq 0$, that is negative semi-definite.

The observer is then:
\[ L \dot{x}_1 = -(1 - u_{av}) \dot{x}_2 + E \]
\[ C \dot{x}_2 = (1 - u_{av}) \dot{x}_1 - \frac{1}{R} \dot{x}_2 + k_2 (x_2 - \dot{x}_2) \]

According to the gotten results from the above graphs, the convergence of the estimated current towards its true value is perfectly achieved.
3.5.3 Buck-Boost converter

a. Passivity based control

The Buck-Boost average model:

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{x} = \begin{bmatrix}
0 & (1-u_{av}) \\
-(1-u_{av}) & 0
\end{bmatrix}x - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix}x + \begin{bmatrix}
E \\
0
\end{bmatrix}u_{av}
\]

The desired voltage is \( \bar{v}_2 = EV_d \), then \( \bar{v}_1 = \frac{E}{R}V_d(v_d - 1) \) and \( \bar{v}_{av} = \frac{V_d}{V_d - 1} \).

The error dynamic can be written as follows:

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{e} = \begin{bmatrix}
0 & (1-u_{av}) \\
-(1-u_{av}) & 0
\end{bmatrix}e - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix}e + \begin{bmatrix}
(1-V_d)E \\
\frac{E}{R}V_d(V_d - 1)
\end{bmatrix}e_u
\]

\( e_u = \left(1 - V_d\right)E \frac{E}{R}V_d(V_d - 1) \)

The proposed control law:

\( e_u = -\gamma e_y = -\gamma \left(1 - V_d\right)E \frac{E}{R}V_d(V_d - 1) e = -\gamma \left(1 - V_d\right)Ee_1 - \gamma \frac{E}{R}V_d(V_d - 1)e_2 \)

The closed loop dynamic will be then:

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{e} = \begin{bmatrix}
0 & (1-u_{av}) \\
-(1-u_{av}) & 0
\end{bmatrix}e - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix}e + \gamma \begin{bmatrix}
(1-V_d)E \\
\frac{E}{R}V_d(V_d - 1)
\end{bmatrix}\begin{bmatrix}(1-V_d)E & \frac{E}{R}V_d(V_d - 1)\end{bmatrix}e
\]

Since the matrix

\[
\begin{bmatrix}
0 & 0 \\
\frac{1}{R} & \gamma \frac{E}{R}V_d(V_d - 1)
\end{bmatrix} + \gamma \begin{bmatrix}
(1-V_d)E \\
\frac{E}{R}V_d(V_d - 1)
\end{bmatrix}\begin{bmatrix}(1-V_d)E & \frac{E}{R}V_d(V_d - 1)\end{bmatrix} = \begin{bmatrix}
\gamma V_d^2 E^2 & -\gamma \frac{E^2}{R}V_d^3 \\
-\gamma \frac{E^2}{R}V_d^3 & 1 + \gamma \frac{E^2}{R^2}V_d^4
\end{bmatrix} > 0
\]

Is positive definite, the dissipativity matching condition is verified.

The control law is as next:

\( u_{av} = \bar{u}_{av} - \gamma e_y = \bar{u}_{av} - \gamma \left(1 - V_d\right)Ee_1 - \gamma \frac{E}{R}V_d(V_d - 1)e_2 \)

Where \( e_1 = x_1 - \frac{E}{R}V_d(v_d - 1) \) et \( e_2 = x_2 - V_dE \)
The passivity based control has positively enhanced the slow open loop response buck-boost converter by making a faster response, removing peaks overruns and no deviations have been occurred.

b. Non-linear observer:

The non-linear observer form

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{\hat{x}} = \begin{bmatrix}
0 & 1 - u_w \\
-(1 - u_w) & 0
\end{bmatrix} \hat{x} - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix} \hat{x} + \begin{bmatrix}
u_wE \\
0
\end{bmatrix} + \begin{bmatrix}k_1 \\
k_2
\end{bmatrix} (x_2 - \hat{x}_2)
\]

The dynamic of the estimation error is given by:

\[
\begin{bmatrix}
L & 0 \\
0 & C
\end{bmatrix}
\dot{\hat{e}} = \begin{bmatrix}
0 & 1 - u_w \\
-(1 - u_w) & 0
\end{bmatrix} \hat{e} - \begin{bmatrix}
0 & 0 \\
0 & \frac{1}{R}
\end{bmatrix} \hat{e} - \begin{bmatrix}k_1 \\
k_2
\end{bmatrix} \hat{e}
\]

with \(\hat{e}^T = [\hat{e}_1 \hat{e}_2], \hat{e}_1 = x_1 - \hat{x}_1, \hat{e}_2 = x_2 - \hat{x}_2\).
Considering the Laypunov function $V = \frac{1}{2} (L\dot{e}_1^2 + C\dot{e}_2^2)$, then $\dot{V} = -k_1\dot{e}_1\dot{e}_2 - (k_2 + \frac{1}{R})\dot{e}_2^2$.

For $k_1 = 0$ and $k_2 > 0$, we get $\dot{V} = -(k_2 + \frac{1}{R})\dot{e}_2^2 \leq 0$, that is negative semi-definite.

The observer is given as follows:

$L\dot{\hat{x}}_1 = (1 - u_{av})\dot{x}_2 + E u_{av}$

$C\dot{\hat{x}}_2 = -(1 - u_{av})\dot{x}_1 - \frac{1}{R}\dot{x}_2 + k_2 (x_2 - \dot{x}_2)$

![Fig. 3.13: Buck-boost non-linear observer.](image)

![Fig. 3.14: Buck-boost closed loop response with non-linear observer.](image)

From the above graphs, it is obviously seen that the convergence of the estimated current towards its true value is a bit achieved, mentioning that the error estimation is not completely zero.
3.5.4 Cuk converter

a. Passivity based control

The Cuk average model:

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2
\end{bmatrix}
\begin{bmatrix}
x \\
x' \\
x'' \\
x'''
\end{bmatrix}
= 
\begin{bmatrix}
0 & -(1 - u_{av}) & 0 & 0 \\
(1 - u_{av}) & 0 & u_{av} & 0 \\
0 & -u_{av} & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
x' \\
x'' \\
x'''
\end{bmatrix} + 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/R
\end{bmatrix}
\begin{bmatrix}
e \\
e' \\
e'' \\
e'''
\end{bmatrix}
\]

If the desired voltage \( \overline{x}_4 = EV_d \), then \( \overline{x}_1 = \frac{E}{R} V_d^2 \), \( \overline{x}_2 = E(1 - V_d) \), \( \overline{x}_3 = \frac{E}{R} V_d \) et \( \overline{\pi}_{av} = \frac{V_d}{V_d - 1} \)

The error dynamic can be written as follows:

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2
\end{bmatrix}
\begin{bmatrix}
e \\
e' \\
e'' \\
e'''
\end{bmatrix}
= 
\begin{bmatrix}
0 & -(1 - u_{av}) & 0 & 0 \\
(1 - u_{av}) & 0 & u_{av} & 0 \\
0 & -u_{av} & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
e \\
e' \\
e'' \\
e'''
\end{bmatrix} + 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/R
\end{bmatrix}
\begin{bmatrix}
e \\
e' \\
e'' \\
e'''
\end{bmatrix}
\]

\[
e_y = \begin{bmatrix} E(1 - V_d) & E R V_d (1 - V_d) & -E(1 - V_d) & 0 \end{bmatrix} e
\]

with \( e_1 = x_1 - \overline{x}_1 = x_1 - \frac{E}{R} V_d^2 \), \( e_2 = x_2 - \overline{x}_2 = x_2 - E(1 - V_d) \), \( e_3 = x_3 - \overline{x}_3 = x_3 - \frac{E}{R} V_d \), \( e_4 = x_4 - \overline{x}_4 = x_4 - V_d E \) and \( e_u = u_{av} - \overline{\pi}_{av} = u_{av} - V_d \ (u_{av} = e_u - \frac{V_d}{1-V_d}) \).

The proposed control law:

\[
e_u = -\gamma e_y = -\gamma \begin{bmatrix} E(1 - V_d) & E R V_d (1 - V_d) & -E(1 - V_d) & 0 \end{bmatrix} e
\]

\[=-\gamma E(1 - V_d) e_1 - \gamma \frac{E}{R} V_d (1 - V_d) e_2 + \gamma E(1 - V_d) e_3\]

The closed loop dynamic will be then:

\[
D \dot{e} = J(u_{av}) e - 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{R}
\end{bmatrix}
\begin{bmatrix}
e \\
e' \\
e'' \\
e'''
\end{bmatrix} + 
\begin{bmatrix}
E(1 - V_d) & E R V_d (1 - V_d) & -E(1 - V_d) & 0 \end{bmatrix}
\begin{bmatrix}
e \\
e' \\
e'' \\
e'''
\end{bmatrix}
\]

Since the matrix
Is positive definite, the dissipativity matching condition is verified.

The control law is given by:

\[
\begin{align*}
\gamma E^2(1-V_d)^2 & \quad \gamma \frac{E^2}{R} V_d (1-V_d)^2 & \quad - \gamma E^2(1-V_d)^2 & \quad 0 \\
\gamma \frac{E^2}{R} V_d (1-V_d)^2 & \quad \gamma (\frac{E}{R} V_d (1-V_d))^2 & \quad - \gamma \frac{E^2}{R} V_d (1-V_d)^2 & \quad 0 \\
- \gamma E^2(1-V_d)^2 & \quad - \lambda \frac{E^2}{R} V_d (1-V_d)^2 & \quad \gamma E^2(1-V_d)^2 & \quad 0 \\
0 & \quad 0 & \quad 0 & \quad \frac{1}{R}
\end{align*}
\]

Figure 3.15: Cuk based passivity control
The closed-loop response of Cuk converter has now the following properties: faster response, 
less oscillations and low start-up overruns peaks, so the passivity control has given a closed 
loop performance with an acceptable efficiency.

Figure 3.16: Cuk closed loop response.

b. Non-linear observer:

The non-linear observer form

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2 \\
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_3 \\
\end{bmatrix}
= 
\begin{bmatrix}
0 & -(1 - u_{av}) & 0 & 0 \\
(1 - u_{av}) & 0 & u_{av} & 0 \\
0 & -u_{av} & 0 & -1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_3 \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
E \\
\dot{e}_1 \\
\dot{e}_2 \\
\dot{e}_3 \\
\end{bmatrix}
+ 
\begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4 \\
\end{bmatrix}
(x_2 - \hat{x}_2)
\]

The dynamic of the estimation error is given by:

\[
\hat{e}^T = \begin{bmatrix}
\hat{e}_1 & \hat{e}_2 & \hat{e}_3 & \hat{e}_4 \\
\end{bmatrix}, \text{ With } \hat{e}_i = x_i - \hat{x}_i, i = 1, 2, 3, 4.
\]

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{e}}_1 \\
\dot{\hat{e}}_2 \\
\dot{\hat{e}}_3 \\
\dot{\hat{e}}_4 \\
\end{bmatrix}
= 
\begin{bmatrix}
0 & -(1 - u_{av}) & 0 & 0 \\
(1 - u_{av}) & 0 & u_{av} & 0 \\
0 & -u_{av} & 0 & -1 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{e}}_1 \\
\dot{\hat{e}}_2 \\
\dot{\hat{e}}_3 \\
\dot{\hat{e}}_4 \\
\end{bmatrix}
+ 
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{e}} \\
\dot{\hat{e}}_1 \\
\dot{\hat{e}}_2 \\
\dot{\hat{e}}_3 \\
\end{bmatrix}
+ 
\begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4 \\
\end{bmatrix}
\begin{bmatrix}
1/R \\
0 \\
0 \\
0 \\
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{e}}_1 \\
\dot{\hat{e}}_2 \\
\dot{\hat{e}}_3 \\
\dot{\hat{e}}_4 \\
\end{bmatrix}
\]

Considering the Lyapunov function

\[
V = \frac{1}{2} (L_1 \hat{e}_1^2 + C_1 \hat{e}_2^2 + L_2 \hat{e}_3^2 + C_2 \hat{e}_4^2)
\]

then
\[
\dot{V} = -(k_1 \dot{e}_1 + k_2 \dot{e}_2 + k_3 \dot{e}_3)\dot{e}_2 - (k_4 \dot{e}_4^2 + \frac{1}{R} \dot{e}_4^2)
\]

If it is supposed that \( k_1 = k_3 = k_4 = 0 \) and \( k_2 > 0 \) makes \( \dot{V} = -(k_4 \dot{e}_4^2 + \frac{1}{R} \dot{e}_4^2) \leq 0 \) is negative semi-definite.

The observer is given as follows:

\[
L_1 \dot{x}_1 = -(1 - u_{wv})\dot{x}_2 + E
\]

\[
C_1 \dot{x}_2 = (1 - u_{wv})\dot{x}_1 + u_{wv}\dot{x}_3 + k_2(x_2 - \dot{x}_2)
\]

\[
L_2 \dot{x}_1 = -u_{wv}\dot{x}_2 - \dot{x}_4
\]

\[
C_2 \dot{x}_2 = \dot{x}_3 - \frac{1}{R} \dot{x}_4
\]

Figure 3.17: Cuk non-linear observer.

Figure 3.18: Cuk closed loop response with non-linear observer.
The convergence of the estimation of the state variables towards their true values is approximately achieved via the nonlinear observer given.

### 3.5.5 Zeta converter

#### a. Passivity based control

The Zeta average model:

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{u}
\end{bmatrix}
=
\begin{bmatrix}
0 & -(1-u_{av}) & 0 & 0 \\
1-u_{av} & 0 & -u_{av} & 0 \\
0 & u_{av} & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
u \\
x \\
u
\end{bmatrix}
+
\begin{bmatrix}
E u_{av} \\
0
\end{bmatrix}
\]

If the desired voltage \( \bar{x}_4 = EV_d \), then \( \bar{x}_1 = \frac{E}{R}V_d^2 \), \( \bar{x}_2 = EV_d \), \( \bar{x}_3 = \frac{E}{R}V_d \) and \( \bar{x}_{av} = \frac{V_d}{V_d + 1} \).

The error dynamic is given by:

\[
L_1 \dot{e}_1 = -(1-u_{av})e_2 + E(V_d + 1)e_u
\]

\[
C_1 \dot{e}_2 = (1-u_{av})e_1 - u_{av}e_3 - \frac{E}{R}V_d(V_d + 1)e_u
\]

\[
L_2 \dot{e}_3 = -u_{av}e_2 - e_4 + E(V_d + 1)e_u
\]

\[
C_2 \dot{e}_4 = e_3 - \frac{1}{R}e_4
\]

this can be written as follows;

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2
\end{bmatrix}
\begin{bmatrix}
\dot{e} \\
\dot{e}_d
\end{bmatrix}
=
\begin{bmatrix}
0 & -(1-u_{av}) & 0 & 0 \\
1-u_{av} & 0 & -u_{av} & 0 \\
0 & u_{av} & 0 & -1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
e \\
e_d
\end{bmatrix}
+
\begin{bmatrix}
E(V_d + 1) \\
0
\end{bmatrix}
\]

\[
e_y = \begin{bmatrix}
E(V_d + 1) \\
-\frac{E}{R}V_d(V_d + 1) \\
E(V_d + 1)
\end{bmatrix} e
\]

The proposed control law is:

\[
e_u = -\gamma e_y = -\gamma \begin{bmatrix}
E(V_d + 1) \\
-\frac{E}{R}V_d(V_d + 1) \\
E(V_d + 1)
\end{bmatrix} e
\]

The closed loop dynamic will be then:
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\[ D\dot{e} = J(u_{av})e - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/R \end{bmatrix} + \gamma \begin{bmatrix} E(V_d + 1) \\ -\frac{E}{R} V_d(V_d + 1) \\ E(V_d + 1) \\ 0 \end{bmatrix} e \]

Evaluating the total derivation of energy function \( V(e) = \frac{1}{2} e^T D e \) that makes:

\[ \dot{V} = -e^T \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/R \end{bmatrix} + \gamma \begin{bmatrix} E(V_d + 1) \\ -\frac{E}{R} V_d(V_d + 1) \\ E(V_d + 1) \\ 0 \end{bmatrix} e \]

Where

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/R \\
\end{bmatrix}
\begin{bmatrix}
E(V_d + 1) \\
-\frac{E}{R} V_d(V_d + 1) \\
E(V_d + 1) \\
0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
1/R \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\gamma E^2 (V_d + 1)^2 & -\gamma \frac{E^2}{R} V_d(V_d + 1)^2 & \gamma E^2 (V_d + 1)^2 & 0 \\
-\gamma \frac{E^2}{R} V_d(V_d + 1)^2 & \gamma \frac{E}{R} V_d(V_d + 1)^2 & -\gamma \frac{E^2}{R} V_d(V_d + 1)^2 & 0 \\
\gamma E^2 (V_d + 1)^2 & -\gamma \frac{E^2}{R} V_d(V_d + 1)^2 & \gamma E^2 (V_d + 1)^2 & 0 \\
0 & 0 & 0 & \frac{1}{R} \\
\end{bmatrix}
\]

Which is semi positive definite so \( V \) is semi negative definite

The control law is given by:

\[
u_{av} = \frac{V_d}{V_d + 1} - \gamma E(V_d + 1)e_1 + \gamma \frac{E}{R} V_d(V_d + 1)e_2 - \gamma E(V_d + 1)e_3
\]

\[
= \frac{V_d}{V_d + 1} - \gamma E(V_d + 1)\left(e_1 - \frac{V_d}{R} e_2 + e_3\right)
\]
The closed-loop response of Zeta converter has now the following properties; faster response, less oscillations and low start-up overruns peaks, so the passivity command has given a closed loop performance with an acceptable efficiency.
b. Non-linear observer:

The non-linear observer form

\[
\begin{bmatrix}
L_i & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix}
= \begin{bmatrix}
0 & -(1-u_{av}) & 0 & 0 \\
(1-u_{av}) & 0 & -u_{av} & 0 \\
0 & u_{av} & 0 & -1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
- \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1/R & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
+ \begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4
\end{bmatrix}(x_4 - \hat{x}_4)
\]

The dynamic of the estimation error is given by:

\[
\begin{bmatrix}
L_i & 0 & 0 & 0 \\
0 & C_1 & 0 & 0 \\
0 & 0 & L_2 & 0 \\
0 & 0 & 0 & C_2
\end{bmatrix}
\begin{bmatrix}
\dot{\hat{e}}_1 \\
\dot{\hat{e}}_2 \\
\dot{\hat{e}}_3 \\
\dot{\hat{e}}_4
\end{bmatrix}
= \begin{bmatrix}
0 & -(1-u_{av}) & 0 & 0 \\
(1-u_{av}) & 0 & -u_{av} & 0 \\
0 & u_{av} & 0 & -1 \\
0 & 0 & 0 & 1/R
\end{bmatrix}
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2 \\
\hat{e}_3 \\
\hat{e}_4
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1/R & 0
\end{bmatrix}
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2 \\
\hat{e}_3 \\
\hat{e}_4
\end{bmatrix}
- \begin{bmatrix}
k_1 \\
k_2 \\
k_3 \\
k_4
\end{bmatrix}
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2 \\
\hat{e}_3 \\
\hat{e}_4
\end{bmatrix}

With: \( \hat{e}^T = [\hat{e}_1 \hat{e}_2 \hat{e}_3 \hat{e}_4] \), With \( \hat{e}_i = x_i - \hat{x}_i, i = 1,2,3,4 \).

Considering the Lyapunov function

\[
V = \frac{1}{2}(L_1 \hat{e}_1^2 + C_1 \hat{e}_2^2 + L_2 \hat{e}_3^2 + C_2 \hat{e}_4^2)
\]

then

\[
\dot{V} = -(k_1 \hat{e}_1 + k_2 \hat{e}_2 + k_3 \hat{e}_3 + k_4 \hat{e}_4) - (k_2 \hat{e}_2^2 + \frac{1}{R} \hat{e}_4^2)
\]

If it is supposed that \( k_1 = k_2 = k_3 = k_4 = 0 \) and \( k_2 > 0 \) makes \( \dot{V} = -(k_2 \hat{e}_2^2 + \frac{1}{R} \hat{e}_4^2) \leq 0 \) is negative semi-definite.

The observer is given as follows:

\[
L_1 \dot{x}_1 = -(1-u_{av})\hat{x}_2 + u_{av}E
\]
\[
C_1 \dot{x}_2 = (1-u_{av})\hat{x}_1 - u_{av}\hat{x}_3
\]
\[
L_2 \dot{x}_3 = u_{av}\hat{x}_2 - \hat{x}_4 + u_{av}E
\]
\[
C_2 \dot{x}_4 = \hat{x}_3 - \frac{1}{R}\hat{x}_4 + k_4(x_4 - \hat{x}_4)
\]
The convergence of the estimation of the state variables towards their true values is approximately achieved via the nonlinear observer given.
3.6 Conclusion:

Another control technique has been demonstrated through this chapter, underlining the advantage using directly the non-linear model of the DC-DC converter, so there is no more need to the linearization for the application of passivity based control approach and its non-linear observer, this approach has proven its efficiency in improving the performance of dc-dc converters, which has a perfect effect on the behavior response by making it faster which means settling time in milliseconds range, and help getting closed loop response with a low peak overruns and lesser deviations.
Conclusion

DC-DC simple PWM converters controllers and linear observers are derived in this humble work, with a proof of controllability for linear state feedback, and observability for state feedback estimation, in addition of passivity based control and non-linear observers are then derived, each with a Lyapunov proof of stability, all these approaches have been used whether to regulate the output voltage or estimate the non-measurable currents. The simulation study well illustrates the theoretical results pointing out a perfect robustness properties of such mentioned controllers. Note that these PWM feedback regulation schemes have been applied on many converter models taking into account the nature of nonlinear parameterization problem of an dc-dc converter, that has been studied within chapter one. The voltage mode control full state feedback by pole placement and PBC work well when the load is constant, in case if the load is changed, the static error is one drawback of the control circuit, means, even though the remarkable effects of those techniques on the converter behavior, by making the converter performance more faster, reduced start-up overruns and less oscillations as possible, still, when it comes to realistic situations the converters are absolutely subject to static disturbance due to load variation, so, when the load is up or down than the proposed one, the control circuit cannot attain the desired output voltage and the difference between the desired and true voltage can be forced to zero by adding an integrator to the average linear control law, i.e. the state variable that appears directly in the output of the converter is introduced as additional integral state variable, which will make sure having an achieved zero static error, unfortunately this will sometimes cause some overshoots and a bit slow settling time as shown within the Boost and Buck-Boost application. The use of linear state observer has been built for reason to sense the non-measurable currents of dc-dc converters, hence, the estimated current and the measured output voltage have been used to elaborate the linear state feedback control law, within steady state, the observers do well estimating the inductor currents, by providing a completely achieved estimated currents, means that the estimation error is fully converged to zero, other than the transient regime of the converter response, it’s an obviously shown that the circuit control has a failed estimation action. As for the non-linear observer based on PBC once shares some properties of the linear observer in what concerns the Zeta converter, means within steady state a perfect estimation is provided and a failed estimation within transient regime, on the other hand this observer is sometimes not efficient as proven within Buck and Boost converters.

So as to conclude, at it has been shown through this piece of work that the dc-dc converters have a high properties after the control application, means that these converters work more better within the closed loop response than the ones within open loop response. Furthermore the techniques control developed during this work could be applied to many other identifiable nonlinear converters. In addition to an advantageous improvement generally good, robustness and simplicity properties additionally convergence of state variables (relying on Linear State Feedback & PBC Nonlinear Observers) are generally achieved.
Bibliography


